Monotone Comparative Statics in the Calvert-Wittman Model *

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Abstract

In this paper, we show that when policy-motivated parties can commit to a particular platform during a uni-dimensional electoral contest where valence issues do not arise there must be a positive association between the policies preferred by candidates and the policies adopted in expectation in the lowest and the highest equilibria of the electoral contest. We also show that this need not be so if the parties cannot commit to a particular policy. The implication is that evidence of a negative relationship between enacted and preferred policies is suggestive of parties that hold positions from which they would like to move from yet are unable to do so.

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1 Introduction

The Downsian model of politics assumes that candidates can commit to keep their policy promises once they reach office. Their ability to commit allows them to manipulate policy proposals so as to garner the fraction of votes that maximizes their probability of winning. Political competition thus leads to convergence of proposed policies to the median voter's ideal point. A number of refinements of this model have been proposed in the literature since Downs's 1957 contribution, many of which have attempted to reverse the problematic hypothesis of complete convergence in policy proposals implied by Downsian competition.[1] Until the late nineties, most of this literature generally took as given the underlying assumption of a perfect capacity of candidates to make credible commitments.[2]

Besley and Coate (1997) and Osborne and Slivinsky (1996), however, showed that some of the key results of the Downsian model no longer hold in a model of citizen-candidates in which policymakers are not bound to keep to their campaign promises. In particular, electoral competition need not lead to full or even partial convergence in policy platforms once candidates lose their ability to make credible promises. Indeed, a multiplicity of equilibria become feasible, some of which entail very extreme policies being proposed in equilibrium.

An extensive literature has developed in the past two decades addressing the issue of how the citizen-candidate assumptions can be reconciled with the intuition of the spatial competition model. These contributions typically model repeated game interactions in which politicians who deviate from their promises are punished in future elections and thus gain an incentive to hold to their campaign promises. (Alesina, 1988; Dixit, Grossman and Gul, 2000; Aragonès, Palfrey and Postlewaite, 2007; Panova, 2017). In some settings, politicians may decide to maintain ambiguity about their preferences either because they do not know the true preferences of the median voter (Glazer, 1990), wish to provide a signal of their character or avoid reputational risks (Kartik and McAfee, 2007; Kartik and van Weelden, 2019). Empirical tests of the credibility hypothesis include comparisons of campaign promises and legislative votes (Sulkin, 2009; Bidwell, Casey and Glennerster 2020), assessments of the effect of term limits on observed policies (Besley and Case, 1995, 2003; Ferraz and Finan, 2011) or

[1] Useful surveys include Mueller (2003), Hinich and Munger(1997), and Roemer (2001).

[2] See also Crain (2004), Chattopadhyay and Duflo (2004), Lee, Moretti and Butler (2004) and Groseclose (2001). For a useful survey of the citizen-candidate model and its dynamic extensions see Duggan and Martinelli (2015).

testing for opportunistic policy cycles (Alesina et al., 1997; Shi and Svennson, 2006).

Investigating whether credibility problems in politics arise and persist is a difficult task, partly because of the problems that the multiplicity of equilibria creates. Our paper sidesteps these problems in uni-dimensional electoral contests where valence issues do not arise by proposing an alternative way of discriminating between models that assume commitment and those that preclude it. We do this by studying the shape of the *expected policy function*, which maps candidates' platforms into expected policies. Optimizing candidates that can make credible commitments will never situate themselves on a segment of that function in which further moderation would drive expected policies to be closer to their ideal points. Doing so would leave unexploited the possibility of increasing the expected payoff for the candidates by driving policies closer to their optimum while increasing their probability of winning the election. This is conceptually similar to the notion that a profit maximizing monopolist would not produce in the decreasing region of its revenue function, as reducing output would then simultaneously increase its revenues and decrease its costs.

Since the platforms chosen by candidates in equilibrium are endogenous, it may be difficult to empirically evaluate hypotheses about the relationship between platforms and policies. In order to address this issue, we show that the *indirect expected policy function,* which maps candidate preferences into observed policies, is also always increasing in the ideal policies of the candidates and can thus be used to investigate whether credibility problems can arise in practice, even in the presence of multiple equilibria.We illustrate how this result can be used to empirically evaluate credibility theories, for example by studying the correlation between changes in constituent or political leaders' preferences as measured by opinion surveys, and enacted policies.

The rest of the paper is organized as follows. Section 2 presents the main results of the paper in detail. Section 3 concludes.

2 Setting

The policy space is the interval $T = [0, 1]$. Voters have ideal policies represented by a point in T. When faced with two policies to choose from, the voter chooses the policy that is closest in distance to the voter's ideal policy.

Candidate preferences are described by a continuous real-valued payoff function $u : T^2 \to \mathbb{R}$: where, for each ideal policy $t \in T$, $u(x,t)$ is strictly concave in platform $x \in T$, with $u(t,t) > u(x,t)$ for all $x \neq t$. There are two candidates, l and r with ideal policies $0 \leq t_l < t_r \leq 1$ who respectively choose platforms x_l and x_r.

Voters' ideal policies are distributed over the policy space T according to a density which is unknown to the candidates. Because of this uncertainty, the policy, $\mathbf{m}$, preferred by the median voter is uncertain and the candidates form beliefs about $\mathbf{m}$ according to a continuous distribution F with full support. Given the profile of platforms (x_l, x_r) proposed by the candidates, the probability of candidate l winning the election is given by:

$$P(x_l, x_r) = \begin{cases} F\left(\frac{x_l + x_r}{2}\right) & \text{if } x_l < x_r \\ \frac{1}{2} & \text{if } x_l = x_r \\ 1 - F\left(\frac{x_l + x_r}{2}\right) & \text{if } x_l > x_r \end{cases}$$

with the probability of r winning the election simply being $1 - P(x_l, x_r)$.

In what follows we sometimes make additional assumptions about the preferences and beliefs of the candidates. We will make it explicit when those additional assumptions are called for.

For $i = l, r$, let $U_{t_i}(x_l, x_r)$ denote the expected payoff function for candidate i with ideal policy t_i, that is,

$$U_{t_i}(x_l, x_r) = P(x_l, x_r)\, u(x_l, t_i) + (1 - P(x_l, x_r))\, u(x_r, t_i).$$

(A1) If $x_l' > x_l \geq t_l$ and $x_r < x_r' \leq t_r$ we have that

$$U_{t_l}(x_l', x_r) \geq U_{t_l}(x_l, x_r) \Rightarrow U_{t_l}(x_l', x_r') > U_{t_l}(x_l, x_r')$$

and

$$U_{t_r}(x_l, x_r') \geq U_{t_r}(x_l, x_r) \Rightarrow U_{t_r}(x_l', x_r') > U_{t_r}(x_l', x_r).$$

That is, the preferences over platforms exhibit a single crossing property in (x_l, x_r). To understand this assumption it helps to understand at this point why candidate i would want to adopt a platform other than t_i. The answer is: in order to decrease the chance that i's opponent wins (which would force candidate i to endure an enacted policy that is far from i's ideal policy, t_i). According to (A1), if it (weakly) pays for candidate i to moderate their platform when the opponent's platform is 'nearby', it definitely pays for candidate i to moderate their platform when the opponent's platform is 'far.' This is so because when the opponent's platform is 'far', it is more painful for candidate i to lose the election.

($A2$) For every $t, t', x, x', y \in T$ with $t < t' \leq x < x' < y$ or $y < x < x' \leq t < t'$

$$\frac{u(x', t') - u(y, t')}{u(x, t') - u(y, t')} > \frac{u(x', t) - u(y, t)}{u(x, t) - u(y, t)}.$$

This is an assumption regarding the log supermodularity in (x, t) of the payoff difference function, $u(x, t) - u(y, t)$ over the set of platforms uniformly to the left, or uniformly to the right, of the platform chosen by the opponent. This says that the relative change in the difference in payoff between winning and losing for a candidate that follows a certain increase in their platform is greater when the candidate's ideal policy is high than when the candidate's ideal policy is low. Examples of functions u that satisfy (A2) include commonly used functions in the literature such as the *quadratic*, $u(x, t) = -(x - t)^2$, the *exponential* $u(x, t) = -e^{(x-t)} + x$, and their positive, affine transformations. See, e.g., Duggan and Martinelli (2017).[3]

In what follows, these assumptions will be employed as in the literature on supermodular games: Assumption (A1) will be used to show that the best response functions of the candidates are increasing in the platform chosen by their opponent, to show that the set of Nash equilibria is non-empty, and to show that this set has a smallest and a largest element. Assumption (A2) will be used to show that the best response functions of the candidates are increasing in their respective ideal policies. Together, both assumptions, and the general structure of our model, imply that the lowest and highest equilibria are increasing in the ideal policies of the candidates, and that therefore the indirect expected policy function is increasing in these ideal policies as well. The reader interested in learning more about these techniques work can consult Amir (2005).

[3]For a different example of an application of log supermodularity in models of politics see Ashworth and Bueno de Mesquita (2006).

2.1 A model with commitment

In this model, as in Calvert (1985) and Wittman (1977), candidate l sets their platform x_l to solve

$$\max_{x_l} U_{t_l}\left(x_l, x_r\right),$$

taking x_r as given.

Candidate r sets their platform x_r to solve

$$\max_{x_r} U_{t_r}\left(x_l, x_r\right),$$

taking x_l as given.

We assume throughout that if two platforms give the same expected payoff for a candidate, given the platform chosen by the opponent, then the candidate will choose the one that is closest to their ideal policy.

In what follows we investigate the characteristics of the Nash equilibria of the game described above.

2.1.1 The best responses of the candidates and their properties

Let $\varphi_i : T \rightrightarrows T$ be the best response correspondence for candidate $i = l, r$.

Lemma 1. *The best response correspondence φ_i for candidate i with ideal policy t_i and platform, x, chosen by i's opponent, is single-valued and has the following properties:*

$$\begin{cases} x < \varphi_{t_i}\left(x\right) \leq t_i & \text{if } x < t_i \\ \varphi_{t_i}\left(x\right) = t_i & \text{if } x = t_i \\ t_i \leq \varphi_{t_i}\left(x\right) < x & \text{if } x > t_i \end{cases}$$

All proofs are in the Appendix.

The interpretation is that candidate i's best response is always 'sandwiched' between t_i and the platform chosen by i's opponent, x.

Proposition 1. *Assume that (A1) holds. Then if $x_r < x_r' \leq t_r$, then we have that $\varphi_{t_l}\left(x_r'\right) \geq \varphi_{t_l}\left(x_r\right)$, and if $x_l' > x_l \geq t_l$, then we have that $\varphi_{t_r}\left(x_l'\right) \geq \varphi_{t_r}\left(x_l\right)$.*

The interpretation is that the best response function for the candidates are non-decreasing in the platforms of their opponent over the set of policies in $\left[t_l, t_r\right]$. Figure 1 illustrates: the solid blue line of the left panel depicts candidate

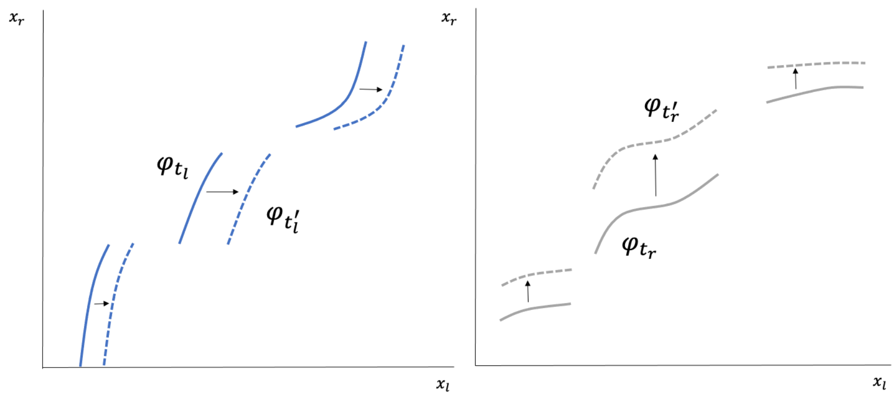

Figure 1: Monotone best response functions

l's best response function, φ_{t_l}, whereas the solid grey line depicts candidate r's best response function, φ_{t_r}.

2.1.2 The Nash equilibria of the game and their properties

Proposition 2. *Assume that (A1) holds. Then the set E of Nash equilibria is non-empty and it has (coordinatewise) largest and smallest elements $(\overline{x}_l^*, \overline{x}_r^*)$ and $(\underline{x}_l^*, \underline{x}_r^*)$.*

Lemma 2. *In equilibrium, $t_l \leq x_l^* < x_r^* \leq t_r$.*

This is the usual 'partial convergence' result one obtains in the Calvert-Wittman model. See, e.g, Roemer (1997), section 4.

2.1.3 Equilibrium Comparative Statics

Theorem 1. *Assume that $(A1)-(A2)$ hold. Then the largest and smallest Nash equilibria of the game, $(\overline{x}_l^*, \overline{x}_r^*)$ and $(\underline{x}_l^*, \underline{x}_r^*)$, are increasing in t_l and t_r.*

To show this result we first establish that the best response functions are increasing in t_l and t_r. This is illustrated in Figure 1: the dashed blue line of the left panel of Figure 1 represents φ_{t_l}, and it is to the right of the solid blue line, which represents φ_{t_l}. In turn, the dashed gray line of the right panel of Figure 1 is the best resp above the solid gray line (the original best response

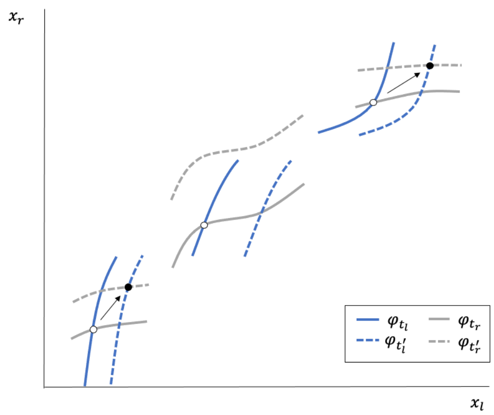

Figure 2: Multiple equilibria comparative statics

function for candidate r). Figure 2 puts all these best response functions together and illustrates the content of Theorem 1: the smallest equilibria of the model parametrized by (t_l, t_r) is smaller than the smallest equilibria of the model parametrized by (t_l', t_r'), when $(t_l', t_r') > (t_l, t_r)$. Similarly for the largest equilibria of the models. Figure 2 makes it clear that comparison of the rest of the equilibria may not even be meaningful, since the model parametrized by (t_l, t_r) has an "intermediate" equilibrium but the model parametrized by (t_l', t_r') does not.

Consider now the expected policy function,

$$\pi(x_l, x_r) := P(x_l, x_r)\, x_l + (1 - P(x_l, x_r))\, x_r.$$

The expected policy function estimates, before the resolution of uncertainty about the electoral outcome, the platform that will ultimately be adopted as policy. The left panel of Figure 3 plots the expected policy as a function of the platform, x_i, chosen by candidate i, given the platform, x, chosen by i's opponent. When $x_i = 0$, i's platform is too extreme to entail a substantial probability of the candidate winning the election, and the expected policy is therefore close to the platform chosen by i's opponent, x. As candidate i moderates their platform, starting from zero, i's probability of winning increases, and the expected policy therefore moves away from x. Eventually, as x_i gets

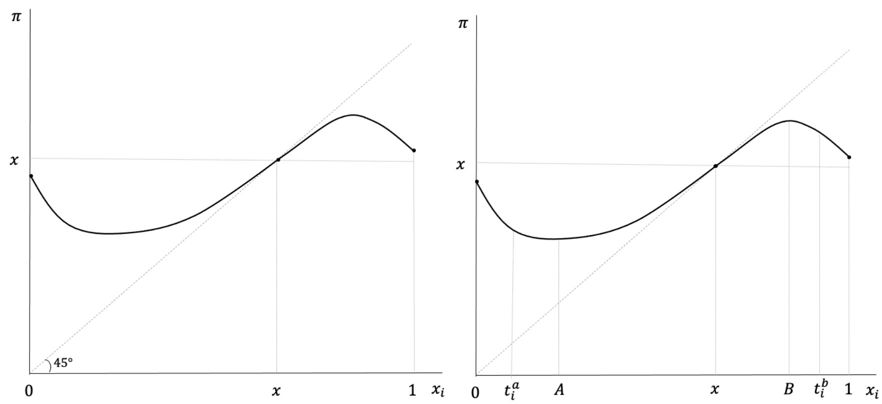

Figure 3: The Expected Policy Function

close to x, so does the expected policy. Similarly, if $x_i = 1$ and this platform is too extreme to entail any substantial probability of candidate i winning the election, the expected policy is close to the platform chosen by i's opponent, x. As candidate i moderates their platform, starting from one, i's probability of winning grows, and the expected policy then begins to move away from x.

Theorem 2. *Let* $x_r > t_l$, $x_l = \varphi_{t_l}(x_r)$ and $x_l' > x_l$. *Then* $\pi(x_l', x_r) > \pi(x_l, x_r)$. *Let* $x_l < t_r$, $x_r = \varphi_{t_r}(x_l)$ and $x_r' < x_r$. *Then* $\pi(x_l, x_r') < \pi(x_l, x_r)$.

Theorem 2 contains the main insight of the paper: a rational candidate would not select a platform that is in the non-increasing region of the expected policy function. The right panel of Figure 3 illustrates this. If t_i is in the increasing region of the expected policy function, the result follows since Lemma 1 shows that $\varphi_{t_i}(x)$is between t_i and x. Now suppose that candidate i's ideal policy is, say t_i^a (resp. t_i^b). Then selecting a platform between t_i^a and A (resp. between B and t_i^b) would leave unexploited the possibility of increasing the expected payoff for the candidate by moderating their platform, as this would drive the expected policy closer to candidate i's ideal point policy while at the same increasing the candidate's probability of winning the election.

Since the platforms chosen by candidates in equilibrium are endogenous, hypotheses testing that relies on direct estimation of the shape of the expected policy function may be riddled with simultaneity bias. In order to address this issue, we note that the *indirect expected policy function,* which maps candidate preferences, an exogenous variable in our model, into observed policies, shares the same comparative statics implications of the expected policy function and can thus be used to investigate whether credibility problems can arise in practice, even in the presence of multiple equilibria.

The *indirect expected policy function* can be computed as follows:

If $(x_l^*, x_r^*) \in E$, then

$$\pi^*(t_l, t_r) = \pi(x_l^*(t_l, t_r), x_r^*(t_l, t_r)).$$

Let $\overline{\pi}^*(t_l, t_r)$ and $\underline{\pi}^*(t_l, t_r)$ be the indirect expected policy corresponding to the largest and smallest equilibrium in E, respectively.

We know from Theorem 2 that in equilibrium the expected policy is increasing in x_l and x_r. From Theorem 1 we know that the largest and smallest Nash equilibria of the game, $(\overline{x}_l, \overline{x}_r)$ and $(\underline{x}_l, \underline{x}_r)$, are increasing in t_l and t_r. It thus follows that the indirect expected policy function associated with the largest and smallest equilibria is also increasing in t_l and t_r. This is the main comparative statics result of the paper, which we summarize below.

Corollary 1. *Assume that* $(A1)-(A2)$ *hold. If* $t_l' > t_l$ *then* $\overline{\pi}^*(t_l', t_r) \geq \overline{\pi}^*(t_l, t_r)$ *and* $\underline{\pi}^*(t_l', t_r) \geq \underline{\pi}^*(t_l, t_r)$. *If* $t_r' < t_r$ *then* $\overline{\pi}^*(t_l, t_r') \leq \overline{\pi}^*(t_l, t_r)$ *and* $\underline{\pi}^*(t_l, t_r') \leq \underline{\pi}^*(t_l, t_r)$.

2.2 A model without commitment

When candidates cannot precommit to adopt a particular platform, voters expect that, if elected, a candidate will implement their most preferred policy once in office. Therefore, the candidates cannot affect the probabilities of being elected and in the unique equilibrium, $x_l^* = t_l$ and $x_r^* = t_r$.[4] Because of this, the adopted platforms are trivially increasing in t_l and t_r.

It turns out, however, that in the model without commitment, Theorem 2 fails and hence the indirect expected policy function *need not be* increasing in the ideal policies of the politicians, as in the model with commitment. We illustrate that this is the case with an example.

Consider a situation where candidates form beliefs about the policy preferred by the median voter, $\mathbf{m}$, as follows: $\mathbf{m}$ is a random variable that is distributed

[4]Because of politicians' inability to make credible commitments, their expected payoffs are unaffected by the choice of platform and they therefore choose the platform that is closest to their ideal policy.

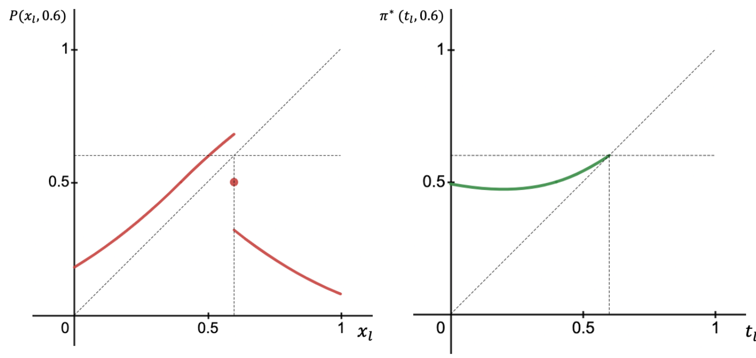

Figure 4: The Model Without Commitment

according to a triangular distribution in the [0,1] interval, with mode 0.5. We also let $u(x,t) = -(x-t)^2$ with $x_r > 0.5$, although nothing in the example depends on these choices.[5] We then investigate the behavior of $P(x_l, x_r)$ as x_l varies given a fixed value of x_r, and of $\pi^*(t_l, t_r)$ as t_l varies given a fixed value of t_r. We obtain that

$$P(x_l, x_r) = \begin{cases} 2\left(\frac{x_l+x_r}{2}\right)^2 & \text{if} \quad x_l \leq 1 - x_r \\ 1 - 2\left(1 - \frac{x_l+x_r}{2}\right)^2 & \text{if} \quad 1 - x_r < x_l < x_r \\ \frac{1}{2} & \text{if} \quad x_l = x_r \\ 2\left(1 - \frac{x_l+x_r}{2}\right)^2 & \text{if} \quad x_l > x_r \end{cases}.$$

The left panel of Figure 4 represents the behavior of $P(x_l, x_r)$ given $x_r = 0.6$, and as x_l varies from zero to one. As expected, the probability of candidate l winning the election grows as the candidate's ideal policy approaches $x_r = 0.6$ from either side, and this probability jumps to 0.5 when both candidates have the same ideal policies.

The indirect expected policy function in this case, when $x_l^* = t_l$ and $x_r^* = t_r$,

[5]The example can be built with any probability distribution over **m** such that $xf(x) > F(x)$ for some value of x. Distributions with these characteristics abound and include, for example, many instances from the Beta and Power families. The example can also be built using Roemer's error distribution model of uncertainty (Roemer 2001, section 2.3).

becomes:

$$\pi^*(t_l, t_r) = \begin{cases} 2\left(\frac{t_l+t_r}{2}\right)^2 \cdot t_l + \left[1 - 2\left(\frac{t_l+t_r}{2}\right)^2\right] \cdot t_r & \text{if } t_l \leq 1 - t_r \\ \left[1 - 2\left(1 - \frac{t_l+t_r}{2}\right)^2\right] \cdot t_l + 2\left(1 - \frac{t_l+t_r}{2}\right)^2 \cdot t_r & \text{if } 1 - t_r < t_l < t_r \\ t_r & \text{if } t_l = t_r \\ 2\left(1 - \frac{t_l+t_r}{2}\right)^2 \cdot t_l + \left[1 - 2\left(1 - \frac{t_l+t_r}{2}\right)^2\right] \cdot t_r & \text{if } t_l > t_r \end{cases},$$

which is a decreasing function of t_l when evaluating the function at any $t_l < \frac{t_r}{3}$. To see this, notice that, when $t_l < 1 - t_r$,

$$\frac{d\pi^*(t_l, t_r)}{dt_l} = \frac{1}{2}\left(3t_l^2 + 2t_l t_r - t_r^2\right).$$

We obtain that $\frac{d\pi^*(0,t_r)}{dt_l} = -t_r^2 < 0$,and $\frac{d^2\pi^*(t_l,t_r)}{dt_l^2} = 3t_l + t_r > 0$. Therefore, as t_l grows from zero, $\frac{d\pi^*(t_l,t_r)}{dt_l}$ becomes less negative, until it reaches zero, when $t_l = \frac{t_r}{3}$, which is the only positive root of $3t_l^2 + 2t_l t_r - t_r^2$.

Hence, if the ideal point for candidate l happens to be to the left of $\frac{t_r}{3}$, the indirect expected policy function will be decreasing in t_l at that point. The right panel of Figure 4 represents the behavior of $\pi^*(t_l, t_r)$ given $t_r = 0.6$, and as t_l varies from zero to 0.6. The expected policy drops as candidate l's ideal policy approaches 0.2, as explained above, and subsequently rises as candidate l's ideal policy grows beyond 0.2, and all the way up to 0.6.

3 Conclusions

We have shown that when candidates can commit to a particular platform during a uni-dimensional electoral contest where valence issues do not arise there must be a positive association between the policies we can expect will be adopted in (the smallest and the largest) equilibrium and the preferred policies held by the candidates. We have also shown that this need not be so if the candidates cannot commit to a particular policy. The implication is that evidence of a negative relationship between enacted and preferred policies in the data is suggestive of candidates that hold positions from which they would like to move from yet are unable to do so. This is the main result of the paper.

This approach can be extended to other models of policy location. For example, Groseclose (2001) proposed a model in which a difference in valence can lead candidates to assume extreme positions. Non-trivial valence differences would violate our symmetry and – under the Groseclose conditions – our monotonicity assumptions, so the approach taken in Section 2 is not directly suitable for testing a valence model. Future research could then focus on (i) allowing for valence and multidimensional issues to play a role, and (ii) understanding what assumptions on the beliefs held by the candidates about the distribution of voter preferences, in lieu of A1-A2, would allow our approach to equilibrium existence and comparative statics to be applicable in these cases.

Our results suggest that empirical work on testing for the existence of credibility problems in politics could advance through direct estimation of the direct and indirect expected policy functions. A regression of enacted policies on policy platforms could shed light on whether the observed correlation between these is positive, as suggested by models of commitment, or negative, as would be the case in citizen-candidate environments. In order to address simultaneity problems in the estimation of the expected policy function, platforms could be instrumented on measures of policymaker or constituent preferences drawn from public opinion surveys, in effect helping us recover the indirect expected policy function.

Anectodally, examples of candidates whose platforms around a single issue were simply too extreme for their own good abound (e.g., George McGovern in 1972 against Richard Nixon and Mario Vargas Llosa in 1990 against Alberto Fujimori). A conventional analysis of the behavior of these politicians would characterize the behavior as relying on gross miscalculations, based on mistaken beliefs about what voters' actual preferences really were. Under the alternative interpretation that we espouse, there is nothing irrational about these policy platforms. It wasn't the policy platforms of these politicians that cost them the elections: it was their preferences. Had they proposed more moderate platforms, voters would not have bought it. The presumption that these politicians do not understand the political environment in which they operate is not needed to explain how we see these politicians behaving during election time.

4 References

Alesina, Alberto (1988), "Credibility and Policy Convergence in a Two-candidate System with Rational Voters," American Economic Review 78, 796-806.

Alesina, Alberto; Nouriel Roubini and Gerald Cohen (1997). Political Cycles and the Macroeconomy. Berkeley: University of California Press.

Enriqueta Aragonès, Andrew Postlewaite, Thomas Palfrey (2007), Political Reputations and Campaign Promises, Journal of the European Economic Association 5(4), 846-884.

Amir, Rabah (2005), "Supermodularity and complementarity in Economics: An Elementary Survey," Southern Economic Journal 71 (3), 636-660.

Ashworth, Scott and Ethan Bueno de Mesquita (2006), "Monotone Comparative Statics and Models of Politics," American Journal of Political Science 50, 214-231.

Bagnoli, Mark and Ted Bergstrom (2005), "Log-concave probability and its applications," Economic Theory 26, 445-469.

Besley, Timothy and Anne Case (1995), "Does Electoral Accountability Affect Economic Policy Choices? Evidence from Gubernatorial Term Limits." Quarterly Journal of Economics 110 (3), 769-798.

Besley, Timothy and Anne Case (2003), "Political Institutions and Policy Choices: Evidence from the United States." Journal of Economic Literature 41(1), 7-73.

Besley, Timothy and Stephen Coate (1997), "An Economic Model of Representative Democracy," Quarterly Journal of Economics 112, 85-114.

Bidwell, Kelly, Katherine Casey and Rachel Glennerster (2020), "Debates: Voting and Expenditure Responses to Political Communication." Journal of Political Economy 128(8), 2880-2924.

Calvert, Randall (1985), "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence," American Journal of Political Science 29, 69-95.

Crain, Mark. (2004), "Dynamic Inconsistency," in C. Rowley and F. Schneider (eds.), Encyclopaedia of Public Choice, 481-484.

Chattopadhyay, Raghabendra and Esther Duflo (2004), "Women as Policy Makers: Evidence from a Randomized Policy Experiment in India," Econometrica 72(5), 1409-1443.

Dixit, Avinash, et al. (2000) "The Dynamics of Political Compromise." Journal of Political Economy 108(3), 531-568.

Duggan, John and César Martinelli (2017), "The Political Economy of Dynamic Elections: Accountability, Commitment and Responsiveness," Journal of Economic Literature 55, 916-984.

Ferraz, Claudio, and Frederico Finan (2011), "Electoral Accountability and Corruption: Evidence from the Audits of Local Governments," American Economic Review 101(4), 1274-1311.

Glazer, Amihai (1990), "The Strategy of Candidate Ambiguity," The American Political Science Review 84(1), 237-241.

Groseclose, Tim (2001), "A Model of Candidate Location When One Candidate Has a Valence Advantage," American Journal of Political Science 45(4), 862-886.

Hinich, Melvin J. and Michael C. Munger (1997), Analytical Politics. Cambridge: Cambridge University Press.

Kartik, Navin and Preston McAfee, R. P. (2007), "Signaling Character in Electoral Competition", American Economic Review 97, 852-870.

Navin Kartik and Richard Van Weelden (2018), "Informative Cheap Talk in Elections," The Review of Economic Studies 86 (2),755-784.

Lee, David. S., Enrico Moretti and Butler, M. J. (2004), "Do Voters Affect or Elect Policies? Evidence from the U.S. House," Quarterly Journal of Economics 119(3), 807-859.

Mueller, Dennis (2003), Public Choice III. Cambridge: Cambridge University Press.

Osborne, Martin and Al Slivinsky (1996), "A model of Political Competition with Citizen-Candidates," Quarterly Journal of Economics 111, 64-96.

Panova, Elena (2016), Partially Revealing Campaign Promises. Journal of Public Economic Theory 19(2), 312-330.

Roemer, John (1997), "Political Economic Equilibrium When Parties Represent Constituents: The Unidimensional Case," Social Choice and Welfare 14, 479-502.

Roemer, John (2001), Political Competition, Cambridge: Harvard University Press.

Shi, Min, and Svensson, Jakob (2006), "Political budget cycles: Do they differ across countries and why?," Journal of Public Economics 90, 1367-1389.

Sulkin, Tracy (2009), "Campaign Appeals and Legislative Action", Journal of Politics 71, 1093–1108.

Vives, Xavier (2001), Oligopoly Pricing: Old Ideas and New Tools, MIT Press.

Wittman, Donald (1977), "Candidates with Policy Preferences: A Dynamic Model," Journal of Economic Theory 14, 180-189.

5 Appendix

Claim 1. *The function $P(\pi_l, \pi_r)$ satisfies the following properties:*

Property (S): For every $x_l, x_r \in T$, $P(x_l, x_r) = 1 - P(x_r, x_l)$.

Property (M): For every $x_l, x_l', x_r \in T$ with $x_l < x_l' < x_r$, $P(x_l, x_r) < P(x_l', x_r)$ and for every $x_l, x_l', x_r \in T$ with $x_r < x_l < x_l'$, $P(x_l, x_r) > P(x_l', x_r)$.

Property (Po): For every $x_l, x_r \in T$, $0 < P(x_l, x_r) < 1$.

Proof. First consider property (S). Let $x_l, x_r \in T$. Then

$$1 - P(x_l, x_r) = \begin{cases} 1 - F\left(\frac{x_l + x_r}{2}\right) & \text{if } x_l < x_r \\ \frac{1}{2} & \text{if } x_l = x_r \\ F\left(\frac{x_l + x_r}{2}\right) & \text{if } x_l > x_r \end{cases}$$

$$= P(x_r, x_l),$$

which is what we wanted to show.

Now consider Property (M).

Let $x_l, x_l', x_r \in T$ with $x_l < x_l' < x_r$. Then $P(x_l', x_r) = F\left(\frac{x_l' + x_r}{2}\right) > F\left(\frac{x_l + x_r}{2}\right) = P(x_l, x_r)$, since F is increasing. Now let $x_l, x_l', x_r \in T$ with $x_r < x_l < x_l'$. Again, since F is increasing,

$$P(x_l', x_r) = 1 - F\left(\frac{x_l' + x_r}{2}\right) < 1 - F\left(\frac{x_l + x_r}{2}\right) = P(x_l, x_r).$$

Now consider Property (Po). If $x_l = x_r$, then the result follows, since $P(x_r, x_l) = 1/2$. Now let $x_l \neq x_r$. Then the result follows since F has full support, which means that, no matter the values of x_l and x_r, there is a positive probability that **m** lies in the interval $\left(0, \frac{x_l + x_r}{2}\right)$ and in the interval $\left(\frac{x_l + x_r}{2}, 1\right)$. □

Proof of Lemma 1

After rearranging terms and eliminating constants that do not depend on candidate l's choice, the candidate's decision problem simplifies to:

$$\max_{x_l} P(x_l, x_r)(u(x_l, t_l) - u(x_r, t_l)).$$

This function is discontinuous at $x_l = x_r$, except when $F(x_r) = 0.5$.

Let $x < t_l$.

First, notice that, for every $x_l \in [0, x)$, $P(x_l, x)(u(x_l, t_l) - u(x, t_l)) < 0 = P(x, x)(u(x, t_l) - u(x, t_l))$. This shows that $\varphi_{t_l}(x) \geq x$. Next, notice that for any $x_l \in (x, t_l]$, we have that

$$P(x_l, x)(u(x_l, t_l) - u(x, t_l)) > 0 = P(x, x)(u(x, t_l) - u(x, t_l)).$$

This shows that $\varphi_{t_l}(x) > x$. Now notice that, for every $x_l \in (t_l, 1]$,

$$P(t_l, x)(u(t_l, t_l) - u(x, t_l)) > P(x_l, x)(u(x_l, t_l) - u(x, t_l)).$$

This is so because, by Property (M), $P(t_l, x) > P(x_l, x)$ and $u(t_l, t_l) > u(x_l, t_l)$. This shows that $\varphi_{t_l}(x) \leq t_l$. We have thus shown that if $x < t_l$ then $x < \varphi_{t_l}(x) \leq t_l$. The correspondence $\varphi_l(x)$ is single-valued when $x < t_l$ since no two different policies x_l and x_l' can at the same time belong to the interval (x, t_l) and be equidistant to t_l.

Let $x = t_l$.

Then clearly $\varphi_{t_l}(x) = t_l$.

Now let $x > t_l$.

As before, notice that, for every $x_l \in (x, 1]$,

$$P(x_l, x)(u(x_l, t_l) - u(x, t_l)) < 0 = P(x, x)(u(x, t_l) - u(x, t_l)).$$

This shows that $\varphi_{t_l}(x) \leq x$. Next, notice that for any $x_l \in [t_l, x)$ we have that $P(x_l, x)(u(x_l, t_l) - u(x, t_l)) > 0 = P(x, x)(u(x, t_l) - u(x, t_l))$. This shows that $\varphi_{t_l}(x) < x$. Now notice that, for every $x_l \in [0, t_l)$,

$$P(t_l, x)(u(t_l, t_l) - u(x, t_l)) > P(x_l, x)(u(x_l, t_l) - u(x, t_l)).$$

This is so because, by Property (M), $P(t_l, x) > P(x_l, x)$ and $u(t_l, t_l) > u(x_l, t_l)$. This shows that $\varphi_{t_l}(x) \geq t_l$. We have thus shown that if $x > t_l$ then $t_l < \varphi_{t_l}(x) \leq x$. The correspondence $\varphi_l(x)$ is single-valued when $x > t_l$ since no two different policies x_l and x_l' can at the same time belong to the interval (t_l, x) and be equidistant to t_l. This completes the proof for φ_{t_l}.

The proof for φ_{t_r} is similar and we omit it here. $\square$

Proof of Proposition 1

Pick x_r and x'_r such that $x'_r > x_r$. Let $x_l = \varphi_{t_l}(x_r)$ and $x'_l = \varphi_{t_l}(x'_r)$. By definition of φ_{t_l},

$$U_{t_l}(x_l, x_r) \geq U_{t_l}(x'_l, x_r).$$

We want to show that $x'_l \geq x_l$.

This has to be so because if $x_l > x'_l$ then, by $(A1)$,

$$U_{t_l}(x_l, x'_r) > U_{t_l}(x'_l, x'_r)$$

which contradicts the fact that x'_l is optimal given x'_r for a candidate with ideal policy t_l. Hence, $\varphi_{t_l}(x'_r) \geq \varphi_{t_l}(x_r)$ for $x'_r > x_r$. The proof for φ_r is similar and we omit it here. □

Proof of Proposition 2

From Proposition 1 we know that the map $(x_l, x_r) \mapsto [\varphi_{t_l}(x_r), \varphi_{t_r}(x_l)]$ is non-decreasing. It follows from Tarski's fixed point theorem that the set $E = \{(x_l, x_r) : (\varphi_{t_l}(x_r), \varphi_{t_r}(x_l)) \geq (x_l, x_r)\}$ is non-empty and forms a complete lattice. Then the set E has greatest element $(\overline{x}_l, \overline{x}_r)$ and a least element $(\underline{x}_l, \underline{x}_r)$. □

Proof of Lemma 2

The proof follows from combining the logical implications of the properties of the best response functions φ_{t_l} and φ_{t_r} identified in Lemma 1.

First, notice there is no equilibrium (x_l, x_r) with $x_r \leq t_l$ or with $x_l < t_l$. To see this, notice that if $x_r \leq t_l$ then $x_r < x_l = \varphi_{t_l}(x_r) \leq t_l$ but then $x_l < x_r = \varphi_{t_r}(x_l) \leq t_r$. On the other hand, $x_l < t_l$ is only a best response for l if $x_r < t_l$, which we just showed cannot arise in equilibrium. Therefore, $x_l < t_l$ also cannot arise in equilibrium.

Similarly, notice there is no equilibrium (x_l, x_r) with $x_l \geq t_l$ or with $x_r > t_r$. To see this, notice that if $x_l \geq t_r$ then $t_r \leq x_r = \varphi_{t_r}(x_l) < x_l$ but then $t_l \leq x_l = \varphi_{t_l}(x_r) < x_r$. On the other hand, $x_r > t_r$ is only a best response for r if $x_l > t_r$, which we just showed cannot arise in equilibrium. Therefore, $x_r > t_r$ also cannot arise in equilibrium.

Then, in equilibrium, $t_l \leq x_l < t_r$ and $t_l < x_r \leq t_r$. It then follows that $t_l \leq x_l = \varphi_{t_l}(x_r) < x_r$.

We have thus established that, in equilibrium, $t_l \leq x_l^* < x_r^* \leq t_r$. □

Claim 2. *Assume that (A2) holds. If* $t_l < t_l' \leq x_l < x_l' < x_r$ *then*

$$U_{t_l}(x_l', x_r) \geq U_{t_l}(x_l, x_r) \Rightarrow U_{t_l'}(x_l', x_r) > U_{t_l'}(x_l, x_r),$$

and if $x_l < x_r < x_r' \leq t_r < t_r'$ *then*

$$U_{t_r}(x_l, x_r') \geq U_{t_r}(x_l, x_r) \Rightarrow U_{t_r'}(x_l, x_r') > U_{t_r'}(x_l, x_r).$$

Proof. Let $t_l < t_l' \leq x_l < x_l' < x_r$.

Assume that $U_{t_l}(x_l', x_r) \geq U_{t_l}(x_l, x_r)$. By the definition of U,

$$P(x_l', x_r)\, u(x_l', t_l) + (1 - P(x_l', x_r))\, u(x_r, t_l) \geq$$

$$P(x_l, x_r)\, u(x_l, t_l) + (1 - P_l(x_l, x_r))\, u(x_r, t_l),$$

which boils down to

$$\frac{P(x_l', x_r)}{P(x_l, x_r)} \frac{[u(x_l', t_l) - u(x_r, t_l)]}{[u(x_l, t_l) - u(x_r, t_l)]} \geq 1.$$

By (A2), we have that

$$\frac{u(x_l', t_l') - u(x_r, t_l')}{u(x_l, t_l') - u(x_r, t_l')} > \frac{u(x_l', t_l) - u(x_r, t_l)}{u(x_l, t_l) - u(x_r, t_l)},$$

therefore

$$\frac{P(x_l', x_r)}{P(x_l, x_r)} \frac{[u(x_l', t_l') - u(x_r, t_l')]}{[u(x_l, t_l') - u(x_r, t_l')]} > \frac{P(x_l', x_r)}{P(x_l, x_r)} \frac{[u(x_l', t_l) - u(x_r, t_l)]}{[u(x_l, t_l) - u(x_r, t_l)]},$$

which means that

$$\frac{P(x_l', x_r)}{P(x_l, x_r)} \frac{[u(x_l', t_l') - u(x_r, t_l')]}{[u(x_l, t_l') - u(x_r, t_l')]} > 1,$$

from where it follows that

$$U_{t_l'}(x_l', x_r) > U_{t_l'}(x_l, x_r).$$

Now let $x_l < x_r < x_r' \leq t_r < t_r'$.

Assume that $U_{t_r}(x_l, x_r') \geq U_{t_r}(x_l, x_r)$. By the definition of U,

$$P(x_l, x_r')\, u(x_l, t_r) + (1 - P(x_l, x_r'))\, u(x_r', t_r) \geq$$

$$P(x_l, x_r)\, u(x_l, t_r) + (1 - P_l(x_l, x_r))\, u(x_r, t_r),$$

which boils down to

$$\frac{[1 - P(x_l, x_r')]}{[1 - P(x_l, x_r)]} \frac{u(x_r', t_r) - u(x_l, t_r)}{u(x_r, t_r) - u(x_l, t_r)} \geq 1.$$

By (A2), we have that

$$\frac{u(x_r', t_r') - u(x_l, t_r')}{u(x_r, t_r') - u(x_l, t_r')} > \frac{u(x_r', t_r) - u(x_l, t_r)}{u(x_r, t_r) - u(x_l, t_r)},$$

therefore

$$\frac{[1 - P(x_l, x_r')]}{[1 - P(x_l, x_r)]} \frac{u(x_r', t_r') - u(x_l, t_r')}{u(x_r, t_r') - u(x_l, t_r')} > \frac{[1 - P(x_l, x_r')]}{[1 - P(x_l, x_r)]} \frac{u(x_r', t_r) - u(x_l, t_r)}{u(x_r, t_r) - u(x_l, t_r)},$$

which means that

$$\frac{[1 - P(x_l, x_r')]}{[1 - P(x_l, x_r)]} \frac{u(x_r', t_r') - u(x_l, t_r')}{u(x_r, t_r') - u(x_l, t_r')} > 1,$$

from where it follows that

$$U_{t_r'}(x_l, x_r') > U_{t_r'}(x_l, x_r).$$

□

Proof of Theorem 1

We first show that $\varphi_{t_l}(x_r)$ is increasing in t_l. Pick t_l and t_l' such that $t_l' > t_l$. Let $x_l = \varphi_{t_l}(x_r)$ and $x_l' = \varphi_{t_l'}(x_r)$. By definition of φ_{t_l},

$$U_{t_l}(x_l, x_r) \geq U_{t_l}(x_l', x_r)$$

We want to show that $x_l' \geq x_l$. If $t_l' \geq x_l$ then the result follows, since by Lemma 1, $x_l' \geq t_l'$.

Now assume that $t_l' \in (t_l, x_l)$ and let $x_l' < x_l$. By Lemma 1, $t_l < t_l' \leq x_l' <$

$x_l < x_r$ and by Lemma 2,

$$U_{t_l'}(x_l, x_r) > U_{t_l'}(x_l', x_r)$$

which contradicts the fact that x_l' is optimal given x_r for a candidate with ideal policy t_l'. Hence, $\varphi_{t_l'}(x_r) \geq \varphi_{t_l}(\pi_r)$ for $t_l' > t_l$.

We now show that $\varphi_{t_r}(x_l)$ is increasing in t_r. Pick t_r and t_r' such that $t_r' > t_r$. Let $x_r = \varphi_{t_r}(x_l)$ and $x_r' = \varphi_{t_r'}(x_l)$. By definition of φ_{t_r},

$$U_{t_r}(x_l, x_r) \geq U_{t_r}(x_l, x_r')$$

We want to show that $x_r' \geq x_r$. Assume, to the contrary, that $x_r' < x_r$. By Lemma 1, $x_l < x_r' < x_r \leq t_r < t_l'$. Then, by Lemma 2,

$$U_{t_r'}(x_l, x_r) > U_{t_r'}(x_l, x_r')$$

which contradicts the fact that x_r' is optimal given x_r for a candidate with ideal policy t_r'. Hence, $\varphi_{t_r'}(x_l) \geq \varphi_{t_r'}(x_l)$ for $t_r' > t_r$.

Let $\overline{x}(t_l, t_r) = \sup\{(x_l, x_r) : (\varphi_{t_l}(x_r), \varphi_{t_r}(x_l)) \geq (x_l, x_r)\}$ be the largest fixed point of $(x_l, x_r) \mapsto [\varphi_{t_l}(x_r), \varphi_{t_r}(x_l)]$, and therefore the largest Nash equilibrium of the game. This equilibrium exists, by Proposition 2.

We obtain that $\overline{x}(t_l, t_r)$ is increasing in t_l and t_r since we just showed that $\varphi_{t_l}(x_r)$ and $\varphi_{t_r}(x_l)$ are increasing in t_l and t_r. The case of the smallest equilibrium is analogous. $\square$

Proof of Theorem 2

We first establish the result for changes in x_l while keeping x_r fixed. Let $x_r > t_l$ and $x_l = \varphi_{t_l}(x_r)$. From Lemma 1 we know that $t_l \leq x_l < x_r$. If $x_l' \geq x_r$ the result follows immediately since

$$P(x_l', x_r)\, x_l' + (1 - P(x_l', x_r))\, x_r > P(x_l, x_r)\, x_l + (1 - P(x_l, x_r))\, x_r$$

regardless of the value of $P(x_l', x_r)$ and $P(x_l, x_r)$, which are always positive.

Now let $x_l' \in (x_l, x_r)$. We want to show that $\pi(x_l', x_r) > \pi(x_l, x_r)$.

Assume not, that is, assume that

$$P(x_l', x_r)\, x_l' + (1 - P(x_l', x_r))\, x_r \leq P(x_l, x_r)\, x_l + (1 - P(x_l, x_r))\, x_r.$$

Cancelling and rearranging terms yields

$$\frac{P\left(x_l', x_r\right)}{P\left(x_l, x_r\right)} \geq \frac{x_r - x_l}{x_r - x_l'}.$$

Since $x_l = \varphi_{t_l}\left(x_r\right)$, it follows that

$$P\left(x_l, x_r\right) u\left(x_l, t_l\right) + \left(1 - P\left(x_l, x_r\right)\right) u\left(x_r, t_l\right) \geq$$

$$P\left(x_l', x_r\right) u\left(x_l', t_l\right) + \left(1 - P\left(x_l', x_r\right)\right) u\left(x_r, t_l\right).$$

We obtain that

$$\frac{P\left(x_l', x_r\right)}{P\left(x_l, x_r\right)} \leq \frac{u\left(x_r, t_l\right) - u\left(x_l, t_l\right)}{u\left(x_r, t_l\right) - u\left(x_l', t_l\right)}.$$

Putting these expressions together yields

$$\frac{u\left(x_r, t_l\right) - u\left(x_l, t_l\right)}{u\left(x_r, t_l\right) - u\left(x_l', t_l\right)} \geq \frac{x_r - x_l}{x_r - x_l'},$$

which is equivalent to

$$u\left(x_l', t_l\right) \leq \frac{x_r - x_l'}{x_r - x_l} u\left(x_l, t_l\right) + \left(1 - \frac{x_r - x_l'}{x_r - x_l}\right) u\left(x_r, t_l\right). \tag{1}$$

But $\frac{x_r - x_l'}{x_r - x_l} x_l + \left(1 - \frac{x_r - x_l'}{x_r - x_l}\right) x_r = x_l'$ and $\frac{x_r - x_l'}{x_r - x_l} \in (0, 1)$. Therefore, strict concavity requires that

$$u\left(x_l', t_l\right) > \frac{x_r - x_l'}{x_r - x_l} u\left(x_l, t_l\right) + \left(1 - \frac{x_r - x_l'}{x_r - x_l}\right) u\left(x_r, t_l\right). \tag{2}$$

The contradiction between equations (1) and (2) establishes the first result.

Now we establish the result for changes in x_r while keeping x_l fixed.

Let $x_l < t_r$, $x_r = \varphi_{t_r}\left(x_l\right)$ and $x_r' < x_r$. From Lemma 1 we know that $x_l < x_r' \leq t_r$. If $x_r' \leq x_l$ the result follows immediately since $P\left(x_l, x_r'\right) x_l + \left(1 - P\left(x_l, x_r'\right)\right) x_r' < P\left(x_l, x_r\right) x_l + \left(1 - P\left(x_l, x_r\right)\right) x_r$ regardless of the value of the probabilities $P\left(x_l, x_r'\right)$ and $P\left(x_l, x_r\right)$, which are always positive.

Now let $x_r' \in \left(x_l, x_r\right)$. We want to show that $\pi\left(x_l, x_r'\right) < \pi\left(x_l, x_r\right)$.

Assume not, that is, assume that

$$P\left(x_l, x_r'\right) x_l + \left(1 - P\left(x_l, x_r'\right)\right) x_r' \geq P\left(x_l, x_r\right) x_l + \left(1 - P\left(x_l, x_r\right)\right) x_r.$$

Cancelling and rearranging terms yields

$$\frac{1 - P\left(x_l, x_r'\right)}{1 - P\left(x_l, x_r\right)} \geq \frac{x_r - x_l}{x_r' - x_l}.$$

Since $x_r = \varphi_{t_r}\left(x_l\right)$, it follows that

$$P\left(x_l, x_r\right) u\left(x_l, t_r\right) + \left(1 - P\left(x_l, x_r\right)\right) u\left(x_r, t_r\right) \geq$$

$$P\left(x_l, x_r'\right) u\left(x_l, t_l\right) + \left(1 - P\left(x_l, x_r'\right)\right) u\left(x_r', t_l\right).$$

We obtain that

$$\frac{1 - P\left(x_l, x_r'\right)}{1 - P\left(x_l, x_r\right)} \leq \frac{u\left(x_r, t_r\right) - u\left(x_l, t_r\right)}{u\left(x_r', t_r\right) - u\left(x_l, t_r\right)}.$$

Putting these expressions together yields

$$\frac{u\left(x_r, t_r\right) - u\left(x_l, t_r\right)}{u\left(x_r', t_r\right) - u\left(x_l, t_r\right)} \geq \frac{x_r - x_l}{x_r' - x_l},$$

which is equivalent to

$$u\left(x_r', t_r\right) \leq \frac{x_r' - x_l}{x_r - x_l} u\left(x_r, t_r\right) + \left(1 - \frac{x_r' - x_l}{x_r - x_l}\right) u\left(x_l, t_r\right). \tag{3}$$

But $\frac{x_r'-x_l}{x_r-x_l} x_r' + \left(1 - \frac{x_r'-x_l}{x_r-x_l}\right) x_l = x_r'$ and $\frac{x_r'-x_l}{x_r-x_l} \in (0,1)$. Therefore, strict concavity requires that

$$u\left(x_r', t_r\right) > \frac{x_r' - x_l}{x_r - x_l} u\left(x_r, t_r\right) + \left(1 - \frac{x_r' - x_l}{x_r - x_l}\right) u\left(x_l, t_r\right). \tag{4}$$

The contradiction between equations (3) and (4) establishes the second result.

□