Distorted Trade Barriers: A Dissection of Trade Costs in a “Distorted Gravity” Model*

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Abstract

It is quite common in the trade literature to use iceberg transport costs to repre-
sent variable trade barriers, both tariffs and shipping costs alike. However, in models
with monopolistic competition these are, in fact, not identical trade restrictions. This
difference is not driven by tariff revenue but by how the two trade costs affect firm
profits and the extensive margin. We illustrate these differences in a gravity model à
la Chaney (2008). We show theoretically that trade flows are more elastic with respect
to ad valorem tariffs than transport costs and find a linear relationship between the
esticities with respect to ad valorem tariffs, iceberg transport costs, and fixed market
costs. We empirically validate these results using data on U.S. product-level imports.

JEL classification: F12; F13; F17

Keywords: Gravity; Firm heterogeneity; Monopolistic competition

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1 Introduction

A common approach in the trade literature is to use iceberg transport costs, shipping more than one unit of output to have one unit arrive as a portion “melts” away, to represent variable trade barriers, both tariffs and shipping costs alike. For many models this equivalence is a reasonable assumption; e.g., in models of perfect competition, iceberg transport costs and ad valorem tariffs are equivalent.\textsuperscript{1} In addition, if we are only interested in the intensive margin, then the two trade barriers are equivalent even under monopolistic competition. However, despite the market price being identical under both types of trade barriers in models with monopolistic competition, the level of firm profit is not. This has far reaching implications as the level of profit determines firm entry and exit – the extensive margin.\textsuperscript{2} It should be noted that this difference is not driven by the fact that tariffs generate income in the destination country (through tariff revenue) and iceberg transport costs generate income in the source country (through an implicit transport sector). The difference in firm profits exists because the firm is able to recoup a portion of its losses in transport via its monopolistic power, whereas tariff revenue is completely captured by the domestic government; i.e. iceberg costs are based on quantity while ad valorem tariffs are based on value.

In this paper we take the difference in variable costs seriously and solve a highly tractable gravity model (based on Chaney 2008) with three costs: fixed costs of production, variable costs based on quantity (iceberg), and variable costs based on value (tariff). Doing this allows us to make several observations. The first observation is that there is a linear relationship linking the trade flow elasticities of all three costs. Specifically, we show that the sum of the elasticity with respect to iceberg costs and fixed costs equals the elasticity with respect to

\begin{footnote}{Of course this equivalence hinges on how the researcher deals with the different sources of income either through a transport sector or tariff revenue. The typical approach has been to assume these away.}
\end{footnote}

\begin{footnote}{The different effect on firm profit is shown explicitly in Cole (2011), which has fixed cost heterogeneity with quasi-linear utility and analyzes how iceberg transport costs and ad valorem tariffs affect the mass of varieties and welfare differently. Schröder and Sørensen (2011) additionally illustrate in a Meltiz (2003) type model how tariffs differ from iceberg transport costs. Neither of these papers highlight this difference in a gravity framework. For a welfare analysis on the differences between per-unit trade costs versus iceberg see Sørensen (2012).}
\end{footnote}
tariffs. The second observation is that the elasticity of trade with respect to tariffs is greater (in magnitude) than the elasticity with respect to iceberg transport costs. We provide empirical support for these two results. In addition, we illustrate that the elasticity of substitution matters in a gravity model with heterogeneous firms. That is to say, the heterogeneous firm literature (see Eaton and Kortum 2002 and Chaney 2008 for example) has shown the trade elasticity with respect to variable costs only depends on a parameter governing the variation in the distribution of firm productivity. However, if variable costs are based on value rather than quantity (e.g. ad valorem tariffs), the elasticity of substitution does not get canceled out by adding up the intensive and extensive margin effects.

We apply our model to data on U.S. imports at the 10-digit HS level for the year 2001. We use ad valorem tariff data from John Romalis’s U.S. Tariff Database and calculate the iceberg transport costs from the available import data as reported by the U.S. Census. Since we are not aware of any direct measures of fixed costs of production at the product level, we examine several proxies for fixed costs. Our starting point is the inverse of the elasticity of substitution from Soderbery (2015) which is estimated at the 10-digit HS level using U.S. data giving us product level variation as our other variables. In order to allow for some country level variation as well (our tariffs and transport cost measures vary both across products and countries), we interact the inverse of the elasticity of substitution with four measures from World Bank’s Ease of Doing Business Database: the ease of doing business index, cost to export, time to export, and days to export. Our results are consistent across the five proxies. We are able to empirically confirm that tariff and transport cost elasticities are different, with the tariff elasticity being larger, and that the tariff elasticity is equal to the sum of the transport and fixed cost elasticities. We also show our results are robust across several robustness exercises.

The rest of the paper proceeds as follows. Section 2 sets up the model, while section 3 introduces trade into the model and finds the elasticities of trade flows with respect to

\[\text{For a thorough review of the vast gravity literature see Head and Mayer (2014).}\]
both iceberg transport costs and ad valorem tariffs. In section 4 we examine our predictions empirically and section 5 concludes.

2 Setup

We follow Chaney (2008) (henceforth Chaney) very closely, maintaining the notation and setup, with two main exceptions. First, we allow for an ad valorem tariff, \( s_{ij}^h \), to be charged on goods shipped from country \( i \) to country \( j \) in sector \( h \) where \( t_{ij}^h = 1 + s_{ij}^h > 1 \). Secondly, we allow for the government to sell bonds to the general public in a very specific way. This is a simplifying assumption, but an important one, which we will discuss in greater detail in subsection 2.3.

There are \( N \) potentially asymmetric countries that produce goods using only labor. Country \( n \) has a population of \( L_n \). Consumers in each country maximize utility derived from the consumption of goods from \( H + 1 \) sectors. Sector 0 provides a single freely traded homogeneous good that pins down the wage in country \( n \), \( w_n \). The other \( H \) sectors are made of a continuum of differentiated goods. If a consumer consumes \( q_0 \) units of good 0, and \( q_h(\omega) \) units of each variety \( \omega \) of good \( h \), for all varieties in the set \( \Omega_h \) (determined in equilibrium), she gets a utility \( U \),

\[
U \equiv q_0^{\mu_0} \prod_{h=1}^{H} \left( \int_{\Omega_h} q_h(\omega)^{\sigma_h-1}/\sigma_h \, d\omega \right)^{[\sigma_h/(\sigma_h-1)]\mu_h},
\]

where \( \mu_0 + \sum_{h=1}^{H} \mu_h = 1 \), and where \( \sigma_h > 1 \) is the elasticity of substitution between two varieties of good \( h \).

\footnote{We assume that every country produces a positive amount of \( q_0 \).}
2.1 Trade Barriers and Technology

There are three types of trade barriers, two of which are variable and one is fixed. The two variable trade barriers are tariffs, $t_{ij}^h$, and iceberg transport costs, $\tau_{ij}^h$, while the fixed barrier is given by fixed cost of production, $f_{ij}^h$. Each firm in sector $h$ draws a random unit labor productivity $\varphi$ from a Pareto distribution with shape parameter $\gamma_h$. Following Chaney, we assume the total mass of potential entrants in each sector is proportional to $w_jL_j$. The cost of producing $q$ units of a good and selling them in country $j$ for a firm with productivity $\varphi$ is

$$c_{ij}^h(p, q) = (t_{ij}^h - 1)pq + \frac{\tau_{ij}^h w_i}{\varphi}q + f_{ij}^h$$

and the total revenue for the same firm is $t_{ij}^h pq$. Therefore the profit is

$$\Pi(p, q, \varphi) = t_{ij}^h pq - (t_{ij}^h - 1)pq - \frac{\tau_{ij}^h w_i}{\varphi}q + f_{ij}^h = pq - \frac{\tau_{ij}^h w_i}{\varphi}q + f_{ij}^h,$$

which collapses to the familiar form found in the literature. The tariff does affect profits, but only through $q$, the quantity demanded. As is standard in these models, the price a firm charges is a constant markup over marginal cost and the price a consumer pays is the price the firm charges plus the tariff,

$$p_{ij}^h(\varphi) = \frac{\sigma_h}{(\sigma_h - 1)} \frac{\tau_{ij}^h w_i}{\varphi} \quad \text{Firm Price}$$

$$p_{ij}^h(\varphi) = t_{ij}^h p_{ij}^h(\varphi) = \frac{\sigma_h}{(\sigma_h - 1)} \frac{t_{ij}^h \tau_{ij}^h w_i}{\varphi} \quad \text{Consumer Price}$$

Note that the tariff and transport cost have the same effect on the price paid by consumers. However, a tariff equal to transport cost will result in a lower level of profit which will have an effect on the extensive margin. To see this insert the price given by equation

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5Productivity is distributed over $[1, +\infty)$ according to $P(\hat{\varphi}_h < \varphi) = G_h(\varphi) = 1 - \varphi^{-\gamma_h}$, with $\gamma_h > \sigma_h - 1$.

6For a cleaner equation, we abuse notation and drop the $i, j$ subscripts on $p$ and $q$. This is also consistent with Chaney.
back into the profit function given by equation (3):

$$\Pi(p, q, \varphi) = t_{ij}^h \left[ \frac{\sigma_h}{(\sigma_h - 1)} \frac{\tau_{ij}^h w_i}{\varphi} \right] q - (t_{ij}^h - 1) \left[ \frac{\sigma_h}{(\sigma_h - 1)} \frac{\tau_{ij}^h w_i}{\varphi} \right] q - \frac{\tau_{ij}^h w_i}{\varphi} q + f_{ij}^h,$$

and as an illustrative example let $\frac{w_i}{\varphi} = 100$, $\sigma_h = 2$, and both trade restrictions to be equal, $t_{ij}^h = \tau_{ij}^h = 1.10$. We decompose the firm’s profit for shipping one unit ($q = 1$) in Table 1 while only allowing for one trade barrier at a time. As can be seen, everything is the same except the variable cost under an iceberg specification is less than that of a tariff. The reason stems from when the cost is incurred. Through monopolistic power, the firm is able to recoup a portion of its losses in transport by charging a markup over marginal cost (which includes transport costs), whereas tariff revenue is completely captured by the domestic government.

<table>
<thead>
<tr>
<th>Trade Barrier</th>
<th>Revenue</th>
<th>Variable Cost</th>
<th>Fixed Cost</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iceberg ($t_{ij}^h = 1, \tau_{ij}^h = 1.10$)</td>
<td>220</td>
<td>110</td>
<td>$f_{ij}^h$</td>
<td>$110 - f_{ij}^h$</td>
</tr>
<tr>
<td>Tariff ($t_{ij}^h = 1.10, \tau_{ij}^h = 1$)</td>
<td>220</td>
<td>0.1(200)+100=120</td>
<td>$f_{ij}^h$</td>
<td>$100 - f_{ij}^h$</td>
</tr>
</tbody>
</table>

### 2.2 Demand for Differentiated Goods

The total income spent by workers in country $j$, $Y_j$, is the sum of their labor income ($w_j L_j$) and of the dividends they get from their portfolio ($w_j L_j \pi$), where $\pi$ is the dividend per share of the global mutual fund consisting of aggregated firm profits and government bonds. Tariff inclusive exports from country $i$ to country $j$ in sector $h$, by a firm with labor productivity $\varphi$, are

$$x_{ij}^h(\varphi) = p_{ij}^h(\varphi)d_{ij}^h(\varphi) = \mu_h Y_j \left( \frac{p_{ij}^h(\varphi)}{P_j^h} \right)^{1-\sigma_h}$$

where $P_j^h$ is the ideal price index for good $h$ in country $j$.\(^7\)

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\(^7\)We have chosen to include tariffs in the value of exports to be consistent with the calculations of Chaney with respect to iceberg transport costs. The proofs for a model using a gravity equation both inclusive and exclusive of trade costs is available in an online appendix. Though the elasticities obviously change, our claims do not.
If only those firms above the productivity threshold $\varphi_{kj}$ in country $k$ and sector $h$ export to country $j$, the ideal price index for good $h$ in country $j$, $P_j$, and dividends per share, $\pi$, are defined as

$$P_j^h = \left( \sum_{k=1}^{N} w_k L_k \int_{\varphi_{kj}}^{\infty} \left( \frac{\sigma_h t_h^{\varphi_{kj}} \tau_{ij}^h w_k}{(\sigma_h - 1) \varphi} \right)^{1-\sigma_h} dG_h(\varphi) \right)^{1/(1-\sigma_h)} \quad (7)$$

$$\pi = \frac{\sum_{h=1}^{H} \sum_{k,l=1}^{N} w_k L_k \left( \int_{\varphi_{kl}}^{\infty} \left[ \pi_{kl}^h(\varphi) + b_{kl}^h(\varphi) \right] dG(\varphi) \right)}{\sum_{n=1}^{N} w_n L_n} \quad (8)$$

where

$$\pi_{kl}^h(\varphi) = \left( \frac{1}{(\sigma_h - 1)} \right) \tau_{kl}^h w_k \varphi - f_{kl}^h$$

are the net profits that a firm with productivity $\varphi$ in country $k$ and sector $h$ earns from exporting to country $l$, and

$$b_{kl}^h = \frac{(t_{kl} - 1)p_{kl}^h(\varphi)q_{kl}^h(\varphi)}{t_{kl}\sigma_h} = \left( \frac{(t_{kl} - 1)}{\sigma_h - 1} \right) \frac{\tau_{kl}^h w_k}{\varphi} q_{kl}^h(\varphi) \quad (10)$$

is the return on country bond investments. This government bond activity plays an important simplifying role that needs more explanation, but first note that $b_{kl}^h$ is less than the tariff revenue generated in country $l$ for sector $h$, which is

$$\text{Tariff Revenue} = \frac{(t_{kl} - 1)p_{kl}^h(\varphi)q_{kl}^h(\varphi)}{t_{kl}} = \sigma_h b_{kl}^h$$

since $\sigma_h > 1$. This means that only a specific portion of tariff revenue is returned to consumers through bond returns.

### 2.3 Home Government and Tariff Revenue

It is important to note why the particular treatment of government tariff revenue was chosen. An inherent part of the iceberg transport cost assumption is that output is lost to the
economy whereas tariffs create revenue for the government. This makes comparing the two trade restrictions problematic, particularly since our argument is that tariffs affect the extensive margin differently than typical transport costs regardless of any demand effects driven by tariff revenue. Therefore, we require the government to “redistribute” tariff revenue back to world consumers in a particular way. This is done for two reasons: it allows for a very reasonable point of comparison between the two trade barriers and it maintains the high tractability of Chaney’s model.

Though it is not explicitly modeled with iceberg transport costs in the literature, there is in fact a transport sector that receives income. It takes labor to produce the output which is “lost” in transport and this labor receives a wage. Given the assumption of sector 0 (the numeraire), this wage is identical across sectors. From a worker’s perspective, it doesn’t matter which sector (s)he is employed in, including the numeraire. Therefore, we assume that whatever government income from tariff revenue is not used to pay bond holders is spent on the numeraire.

In addition to their wage, a worker receives income from a global mutual fund that redistributes firm profits. This is a very nice assumption that Chaney uses to get zero profits without having the additional complexities of a free entry condition. Since firm profits are lower with a tariff than an identical iceberg transport cost, dividends from this fund are lower and tractability is severely threatened. Therefore, we assume that governments are active in the bond market and keep a budget that results in a specific level of bond payments described by equation (III). Combining firm profits, (I), with government bond payments, (III), results in the following equation:

\[ r_{ij}(\cdot) = \pi_{kl}^h + b_{kl}^h = \left( \frac{1}{(\sigma_h - 1)} \right) \beta_{kl}(\phi) \frac{w_k^h}{\phi} q_{kl}(\phi) - f_{kl}^h, \]

which is identical to the dividends received in the Chaney model that only included iceberg transport costs. This means that the income, associated with each variety in existence, con-
3 Trade with Heterogeneous Firms

In this section, we characterize the equilibrium with trade. Due to the independence of sectors, we only consider sector $h$ and drop the $h$ superscript. The profits firm $\varphi$ earns when exporting from country $i$ to $j$ are

$$\pi_{ij} = \mu Y_j t_{ij} \left[ \frac{\sigma w_i t_{ij} \tau_{ij}}{(\sigma - 1) \varphi P_j} \right]^{1-\sigma} - f_{ij}. $$

Define the threshold $\bar{\varphi}_{ij}$ from $\pi_{ij}(\bar{\varphi}_{ij}) = 0$ as the productivity of the least productive firm in country $i$ able to export to country $j$:

$$\bar{\varphi}_{ij} = \lambda_1 \left( \frac{f_{ij} t_{ij}}{Y_j} \right)^{\frac{1}{\sigma - 1}} \frac{w_i \tau_{ij}}{P_j} \quad (11)$$

where $\lambda_1 = \left( \frac{\sigma}{\sigma - 1} \right)^{1/(\sigma - 1)}$ is a constant. It is easy to see that tariffs affect the threshold firm differently than iceberg transport costs and that, all else equal, a tariff would correspond to a higher threshold (and productivity) than an identical transport cost.

Recalling that $Y_k = w_k L_k (1 + \pi)$ so $w_k L_k = \frac{Y_k}{(1 + \pi)}$, the price index can be written as

$$P_j = \lambda_2 Y_j^{\frac{(\sigma - 1) - \gamma}{(\sigma - 1)}} \theta_j \quad (12)$$

where

$$\lambda_2 = \left( \frac{\gamma - (\sigma - 1)}{\gamma} \right) \left( \frac{\sigma}{\mu} \right)^{\frac{\gamma - (\sigma - 1)}{(\sigma - 1)}} \left( \frac{\sigma}{(\sigma - 1)} \right)^{\gamma} \left( \frac{1 + \pi}{Y} \right)$$

$$\theta_j^{-\gamma} = \sum_{k=1}^{N} \left( \frac{Y_k}{Y} \right)^{-\gamma} (w_k \tau_{ij})^{-\gamma} f_{kj}^{1+\frac{\alpha}{\sigma}} f_{kj}^{1+\frac{\gamma}{\sigma}}.$$

Note that in order for firm profits to be affected in the same way regardless of the trade barrier, income $Y_j$ would have to be a constant multiple of $t_{ij}$. 

sumers receive is identical regardless of how the modeler chooses to represent trade barriers.
The term $\theta_j$ is a measure of country $j$’s remoteness from the rest of the world. Using the general equilibrium price index, (12), we can solve for firm level exports, the productivity thresholds, and total world profits:

\[
x_{ij}(\varphi) = \begin{cases} 
\lambda_3 \left( \frac{Y_i}{Y_j} \right)^{\frac{(\sigma-1)}{\gamma}} \left( \frac{\theta_j}{w_{ij}t_{ij}} \right)^{\sigma-1} \varphi^{\sigma-1}, & \text{if } \varphi \geq \bar{\varphi}_{ij} \\
0 & \text{otherwise}, \end{cases}
\]

(13)

\[\bar{\varphi}_{ij} = \lambda_4 \left( \frac{Y_i}{Y_j} \right)^{\frac{1}{\gamma}} \left( \frac{w_{ij}t_{ij}}{\theta_j} \right)^{\frac{1}{\sigma-1}} (f_{ij}t^*_{ij})^{\frac{1}{\sigma-1}},\]

\[Y_i = (1 + \lambda_5)w_iL_i\]

\[\pi = \lambda_5\]

where $\lambda_3$, $\lambda_4$, and $\lambda_5$ are constants.

It is important to note how tariffs and transport costs enter into the equilibrium firm level of exports and productivity thresholds. Since the price consumers pay is identical under the two trade costs, the quantity of each variety sold is identical $x_{ij}(\varphi)$ – what changes is the number of varieties, $\bar{\varphi}_{ij}$. This difference translates into the following gravity equation:

Total (trade cost inclusive) exports, $X^h_{ij}$, in sector $h$ from country $i$ to country $j$ are given by

\[X^h_{ij} = \mu_h \left( \frac{Y_i Y_j}{Y} \right)^{\gamma h} \left( \frac{w_{ij}t^*_{ij}h_{ij}}{\theta_j} \right)^{\frac{\sigma_h-1}{\gamma_h}} f_{ij}^{-\frac{1}{\sigma_h-1}} \left[ \varphi_{ij}^{\gamma_h-1} - 1 \right].\]

(14)

Exports are a function of country size ($Y_i$ and $Y_j$), workers’ productivity ($w_i$), the bilateral

\[
\begin{align*}
\lambda_3 &= \sigma \lambda_4^{1-\sigma} \\
\lambda_4 &= \left[ \left( \frac{\sigma}{\mu} \right) \left( \frac{\gamma}{\gamma - (\sigma - 1)} \right) \frac{1}{(1 + \lambda_5)} \right]^{\frac{1}{\gamma}} \\
\lambda_5 &= \frac{\sum_{h=1}^{H} \left( \frac{\sigma_h - 1}{\gamma_h} \right) \mu_h}{1 - \sum_{h=1}^{H} \left( \frac{\sigma_h - 1}{\gamma_h} \right) \mu_h}
\end{align*}
\]
trade costs, variable \((t_{ij}^h, \tau_{ij}^h)\) and fixed \((f_{ij}^h)\), and the measure of \(j\)'s remoteness from the rest of the world \((\theta_j^h)\)\(^\text{10}\). It can easily be seen that tariffs and trade costs enter the gravity function differently which is purely driven by the extensive margin. This point is made more clearly by separating out the trade elasticities into the two margins. We additionally report the elasticity with respect to fixed costs:

\[
\text{Tariff: } \theta = -\frac{d \ln X_{ij}}{d \ln t_{ij}} = (\sigma - 1) + \frac{\sigma \gamma - (\sigma - 1)}{\sigma - 1} = \frac{\sigma \gamma}{\sigma - 1} - 1, \quad (15)
\]

\[
\text{Iceberg: } \zeta = -\frac{d \ln X_{ij}}{d \ln \tau_{ij}} = (\sigma - 1) + [\gamma - (\sigma - 1)] = \gamma, \quad (16)
\]

\[
\text{Fixed Cost: } \xi = -\frac{d \ln X_{ij}}{d \ln f_{ij}} = 0 + \frac{\gamma}{\sigma - 1} - 1 = \frac{\gamma}{\sigma - 1} - 1. \quad (17)
\]

There are three conclusions from these elasticities that warrant particular attention.

**Result 1.** The elasticity with respect to tariffs is equal to the sum of the elasticity with respect to fixed and iceberg costs:

\[
\xi + \zeta = \theta. \quad (18)
\]

Result 1 means that if the researcher believed this model completely and took it to the data, she should test that the estimated coefficients satisfy this restriction. If the restriction is not satisfied then the parameters should be restricted accordingly. The second conclusion is straightforward to see by comparing equation \((15)\) with \((16)\) and follows from Result 1.

\(^{10}\)The proof of equation \((14)\) is available in the appendix. Furthermore, following Chaney, we also assume that country \(i\) is small enough and/or remote enough, so that \(\partial \theta_j/\partial t_{ij} \approx 0\) and \(\partial \theta_j/\partial \tau_{ij} \approx 0\).
Result 2. Trade flows are more elastic with respect to changes in tariffs than transport costs,

\[ \vartheta - \zeta = \frac{\gamma - (\sigma - 1)}{\sigma - 1} > 0. \]  

(19)

The difference between trade elasticities depends on two things: the elasticity of substitution and dispersion of productivity among firms in equilibrium. With respect to the elasticity of substitution, the intuition is as follows: For highly competitive industries where a firm’s markup is quite low, the ability of a firm to recoup some of its transport costs is also lower and thus the wedge between profit values is smaller. The shape parameter of the firms’ productivity distribution also plays an important role. When a sector has a high \( \gamma \), the smaller, less productive firms are producing relatively more of the sector’s output. Since changes in tariffs have a greater impact on whether these firms are producing or not, it will then also have a greater impact on the industry’s aggregate trade flow.

The third conclusion is that the elasticity of substitution (\( \sigma \)) does play a role in the elasticity of trade flows with respect to variable costs (contrary to the broad claim by Chaney and Eaton and Kortum 2002) when the variable cost is a function of product value; e.g. ad valorem tariffs.\(^{11}\) However, the claim by Chaney that the elasticity of trade flows is decreasing in the elasticity of substitution is not only maintained by using tariffs, but is strengthened by it.

Result 3. The elasticity of trade flows with respect to ad valorem tariffs is decreasing in the elasticity of substitution,

\[ \frac{d\vartheta}{d\sigma} = \frac{-\gamma}{(\sigma - 1)^2} < 0. \]  

(20)

It is crucial to point out how important the extensive margin is for the last two results; equations (19) and (20). If, for example, there was no entry and exit in the export sector, the tariff elasticity would be identical to the iceberg trade cost elasticity. Moreover, the

\(^{11}\)Though not explicitly shown in this model, due to the specific treatment of income, it is intuitive that other trade costs such as insurance would have a similar effect.
magnitude would be increasing in the elasticity of substitution which is the prediction of Krugman (1980). Therefore, as we move from the theory to the empirics, the reader should keep in mind where a tractable (and simplified) theoretical model may fail us in the real world. In particular, the theory assumes that the extensive margin is able to react to changes in trade costs in line with the intensive margin. This can fail for various reasons. One such reason is that productivity is distributed by a discrete distribution instead of continuous. In this case, the zero profit condition (equation (11)) becomes a non-negative profit condition and it is possible for the least productive exporting firm to make positive ex post profits and the next firm down in the productivity ladder would make negative profits if it exported. Therefore, a sufficiently small change in trade costs would have no effect on the extensive margin. Another possibility is that there are additional barriers to entry outside of the current model. Finally, in terms of timing, the model assumes firms can enter and exit the foreign market as quickly as an incumbent firm can adjust its production. This is particularly important in a time series model as the effect of the extensive margin would lag behind that of the intensive margin otherwise.

4 Empirical Application

We examine our model empirically using U.S. 10-digit HS imports data sourced from the U.S. Census’ Imports of Merchandize for the year 2001. To conduct an empirical investigation, in addition to trade data, we need three more pieces of information: tariffs, transportation costs, and fixed costs of production. We use John Romalis’s U.S. Tariff Database (Feenstra, Romalis, and Schott 2002) as the source of tariff rates the U.S. assessed in 2001. While the Census data allow us to calculate the average collected tariff as it provides information on duties collected and the dutiable value of imports (as has been done by Besedeš and Prusa 2013, among others), we prefer to use the U.S. Tariff Database as it provides us with actual tariffs the U.S. assesses, the rates which firms react to. We use the product-level value of
imports inclusive of collected duties as well as charges for freight and insurance given our model.\textsuperscript{12} We restrict our sample to products with positive tariffs, as our model applies to instances where trade costs are positive.\textsuperscript{13} We further restrict the sample by dropping observations with unreasonably high transportation costs, which we define as transport costs equal to the value of imports.\textsuperscript{14} The Census data allow us to calculate the ad valorem transport cost for every country-product pair observed in the data. We use the ratio of import charges (all freight, insurance, and other charges exclusive of the tariff charged) and imports as the iceberg-melt factor.

The most difficult data to obtain for our exercise are data on fixed costs of production at the country-product level. We are not familiar with any source of data providing such information, so we resort to several proxies for fixed costs. Assuming constant marginal costs (as is the case in our model following equation \textsuperscript{2}) and increasing returns to scale in production, the elasticity of substitution is directly related to fixed costs, with a lower elasticity implying higher fixed costs. Our first proxy for fixed costs is the inverse of the elasticity of substitution that we source from Soderbery (2015), who provides estimates of the elasticity of substitution at the 10-digit HS level. We use the inverse of the elasticity of substitution so that a higher value corresponds to higher fixed costs. One potential difficulty with respect to using the elasticity of substitution as a proxy for fixed costs is that it only varies across the 10-digit HS product codes, but not countries. In order to introduce variation across countries we interact the inverse of the elasticity of substitution with data from World Bank’s Ease of Doing Business Database which have been used to proxy for fixed costs: the ease of doing business index, cost to export, documents to export, and time to export. We use each measure for the earliest year available which is 2011 for the ease of doing business

\textsuperscript{12}We note that our results are qualitatively unchanged if we use imports exclusive of collected duties and charges for freight and insurance.

\textsuperscript{13}The inclusion of imports of zero-tariff products which face positive transport and fixed costs would bias the elasticity of trade with respect to tariffs downward given there would be a number of observations with a zero tariff. Our results reported below are qualitatively unchanged if we include all zero-tariff observations along with a dummy variable which identifies them.

\textsuperscript{14}There are 931 such observations.
index and 2005 for the remaining three measures. As we will show, our results are not particularly sensitive with respect to any of these four measures.

We estimate a gravity equation using OLS with the log of U.S. imports of product \( h \) from country \( i \) \( (\ln X^h_i) \) as our dependent variable and regress it on the log of tariffs \( (\ln t^h_i) \), transportation costs \( (\ln \tau^h_i) \), and fixed costs \( (\ln f^h_i) \). We include country level fixed effects as a proxy for multilateral resistance terms \( (r_i) \). We estimate

\[
\ln X^h_i = \beta_0 + \beta_1 \ln t^h_i + \beta_2 \ln \tau^h_i + \beta_3 \ln f^h_i + r_i + \epsilon^h_i \tag{21}
\]

where \( |\beta_1| = \vartheta \), \( |\beta_2| = \zeta \), \( |\beta_3| = \xi \), and \( \epsilon^h_i \) is the error term. The results from our basic specification using all available data are shown in Table 2. The last two rows in the table report the p-values resulting from testing whether estimated coefficients satisfy restriction \( (18) \), that the elasticity of trade flows with respect to tariff and transport costs add up to the elasticity with respect to fixed costs, and restriction \( (19) \), that the elasticity with respect to tariffs exceeds the elasticity with respect to transport costs. In the latter case we conduct a test on the equality of the two estimates and report the p-value from the one-sided test.

We first note that our estimated coefficients are largely consistent across the five different proxies of fixed costs we use. Our estimates imply that the elasticity of U.S. imports with respect to tariffs is between -0.471 and -0.480, with respect to transport costs between -0.376 and -0.379, and with respect to fixed costs between -0.037 and -0.040, though the elasticity with respect to fixed costs is not significantly different than zero. However, we are more interested in ascertaining whether the implied restrictions are satisfied than in the size of the estimated coefficients. Let us first focus on Result 1. In every specification we cannot reject the hypothesis that the elasticities with respect to transport costs and fixed costs add up to the elasticity with respect to tariffs. Since we only use a proxy for fixed costs, and not direct measures of fixed costs, our preference is that we test whether the coefficient satisfy the predicted relationship, rather than appropriately constraining estimated coefficients. As
Table 2: Full Sample

<table>
<thead>
<tr>
<th></th>
<th>Fixed cost proxy</th>
<th>Elasticity of substitution interacted with</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Elasticity of substitution</td>
<td>Ease of doing business</td>
</tr>
<tr>
<td>Tariff rate (-θ)</td>
<td>-0.471***</td>
<td>-0.480***</td>
</tr>
<tr>
<td></td>
<td>(0.08038)</td>
<td>(0.08116)</td>
</tr>
<tr>
<td>Transport cost (-ζ)</td>
<td>-0.376***</td>
<td>-0.377***</td>
</tr>
<tr>
<td></td>
<td>(0.03648)</td>
<td>(0.03710)</td>
</tr>
<tr>
<td>Fixed cost (-ξ)</td>
<td>-0.040*</td>
<td>-0.037</td>
</tr>
<tr>
<td></td>
<td>(0.20885)</td>
<td>(0.24390)</td>
</tr>
<tr>
<td>Observations</td>
<td>45,918</td>
<td>45,053</td>
</tr>
<tr>
<td>Number of countries</td>
<td>88</td>
<td>66</td>
</tr>
<tr>
<td>R²</td>
<td>0.058</td>
<td>0.060</td>
</tr>
</tbody>
</table>

Test hypotheses and p-values

Result 1: θ = ζ + ξ

<table>
<thead>
<tr>
<th></th>
<th>Result 1: θ = ζ + ξ</th>
<th>Result 2: θ = ζ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5268</td>
<td>0.1085</td>
</tr>
<tr>
<td></td>
<td>0.4550</td>
<td>0.0957</td>
</tr>
<tr>
<td></td>
<td>0.4801</td>
<td>0.1045</td>
</tr>
<tr>
<td></td>
<td>0.4801</td>
<td>0.1045</td>
</tr>
<tr>
<td></td>
<td>0.4801</td>
<td>0.1045</td>
</tr>
</tbody>
</table>

The dependent variable is log of imports. Country fixed effects are included, robust standard errors clustered on countries are in parenthesis with *, **, *** denoting significance at 10%, 5%, and 1%.

Far as Result 2 is concerned note that in every specification the tariff elasticity is estimated to be larger than the transport cost elasticity (in absolute value). Note however, that only when we proxy for fixed costs using both the elasticity of substitution and ease of doing business data can we reject the null hypothesis of the equality of the two elasticities with some degree of significance (10% level).

As discussed at the end of section it is possible that the extensive margin differs across products with some products having a lot of entry and exit and some having little entry and exit. For the latter group of products, we should observe no differences in tariff and transport cost elasticities, while the difference should be significant for products where there is a lot of activity along the extensive margin. In order to examine the extent to which the extensive margin affects our results we split our sample according to the extent of activity on the extensive margin. We use two definitions of the extensive margin. We first calculate, by each product, the amount of entry and exit in 2001. We then define the extensive margin...
as the total turnover (sum of entry and exit) and as net turnover or net entry (entry–exit). How active must the extensive margin be for the predicted difference to be observed is an empirical question since the model is silent on it. After experimenting with various values, we found that the extensive margin must be very active for the predicted differences to emerge. Under both definitions of the extensive margin, once we reach the 95th in the distribution of extensive margin values we start observing the predicted differences.\footnote{For the total turnover definition, the 95th percentile is entry and exit adding to 26, while for net entry the 95th occurs when net entry is lower than -7 or higher than 5.} Table 3 collects the results from estimating our model using both the high and low extensive margin samples. In order to conserve space, we only present the results for one of the Doing Business variables (Ease of doing business), while noting that the other three generate qualitatively identical results.

It is clear that differences in the extensive margin across products play an important role. Limiting our sample to just those products where there is more activity along the extensive margin (top panel of Table 3), strengthens our results. Not only does the difference between the trade and transport cost elasticities grow, it becomes significant at the 1% level in every specification. In addition, we again cannot reject the hypothesis that the trade elasticity is equal to the sum of the transport cost and fixed cost elasticities. To fully confirm the important role played by the extensive margin, the bottom panel of Table 3 collects the results from estimating our specifications on the remaining sample of products characterized by lower activity along the extensive margin. Note that in this sample Result 1 still holds, but not Result 2, confirming the theoretical prediction that when there is not much activity along the extensive margin the elasticity of trade flows with respect to tariffs equals that with respect to transport costs.

Unfortunately, our data preclude us from examining Result 3 empirically, that the elasticity of trade with respect to tariffs is decreasing in the elasticity of substitution. We use the elasticity of substitution as a proxy for fixed costs giving us product level variation. For this reason we are reluctant to also use it as the elasticity of substitution given the difficulty.
Table 3: The Role of Extensive Margin

<table>
<thead>
<tr>
<th></th>
<th>High extensive margin</th>
<th>Net entry</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total turnover</td>
<td></td>
<td>Elasticity of</td>
<td>Elasticity of</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>substitution</td>
<td>substitution</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ease of doing</td>
<td>Ease of doing</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>business</td>
<td>business</td>
</tr>
<tr>
<td>Tariff rate</td>
<td>-0.861***</td>
<td>-0.869***</td>
<td>-0.729***</td>
<td>-0.730***</td>
</tr>
<tr>
<td></td>
<td>(0.09844)</td>
<td>(0.09984)</td>
<td>(0.09958)</td>
<td>(0.10036)</td>
</tr>
<tr>
<td>Transport cost</td>
<td>-0.392***</td>
<td>-0.401***</td>
<td>-0.356***</td>
<td>-0.360***</td>
</tr>
<tr>
<td></td>
<td>(0.06235)</td>
<td>(0.06354)</td>
<td>(0.04899)</td>
<td>(0.04957)</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>-0.386***</td>
<td>-0.382***</td>
<td>-0.186***</td>
<td>-0.176***</td>
</tr>
<tr>
<td></td>
<td>(0.04117)</td>
<td>(0.04032)</td>
<td>(0.03907)</td>
<td>(0.03932)</td>
</tr>
<tr>
<td>Constant</td>
<td>9.163***</td>
<td>10.377***</td>
<td>9.939***</td>
<td>10.478***</td>
</tr>
<tr>
<td></td>
<td>(0.27887)</td>
<td>(0.30992)</td>
<td>(0.29779)</td>
<td>(0.31117)</td>
</tr>
<tr>
<td>Observations</td>
<td>2,479</td>
<td>2,361</td>
<td>3,170</td>
<td>3,067</td>
</tr>
<tr>
<td>Number of countries</td>
<td>73</td>
<td>60</td>
<td>72</td>
<td>61</td>
</tr>
<tr>
<td>R²</td>
<td>0.153</td>
<td>0.159</td>
<td>0.110</td>
<td>0.112</td>
</tr>
<tr>
<td>Test p-values</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Result 1: $\vartheta = \zeta + \xi$</td>
<td>0.5641</td>
<td>0.5610</td>
<td>0.1399</td>
<td>0.1323</td>
</tr>
<tr>
<td>Result 2: $\vartheta = \zeta$</td>
<td>0.0004</td>
<td>0.0005</td>
<td>0.0012</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

|                          | Low extensive margin  |            | Elasticity of    | Elasticity of    |
|                          |                       |            | substitution     | substitution     |
|                          |                       |            | Ease of doing    | Ease of doing    |
|                          |                       |            | business         | business         |
| Tariff rate              | -0.446***             | -0.456*** | -0.443***        | -0.453***        |
|                         | (0.07910)             | (0.07971) | (0.08074)        | (0.08144)        |
| Transport cost           | -0.371***             | -0.372*** | -0.378***        | -0.379***        |
|                         | (0.03846)             | (0.03909) | (0.03941)        | (0.04010)        |
| Fixed cost               | -0.018                | -0.015    | -0.018           | -0.015           |
|                         | (0.02196)             | (0.02200) | (0.02161)        | (0.02168)        |
| Constant                 | 10.444***             | 10.459*** | 10.435***        | 10.451***        |
|                         | (0.20286)             | (0.23513) | (0.20787)        | (0.23823)        |
| Observations             | 43,439                | 42,692    | 42,748           | 41,986           |
| Number of countries      | 88                    | 66        | 88               | 66               |
| R²                       | 0.055                 | 0.056     | 0.055            | 0.057            |
| Test p-values            |                       |           |                  |                  |
| Result 1: $\vartheta = \zeta + \xi$ | 0.4893 | 0.4072 | 0.5750 | 0.4866 |
| Result 2: $\vartheta = \zeta$ | 0.1514 | 0.1285 | 0.1940 | 0.1674 |

The dependent variable is log of imports. Country fixed effects are included, robust standard errors clustered on countries are in parenthesis with *, **, *** denoting significance at 10%, 5%, and 1%.

that would create in interpreting our results.
4.1 Robustness

4.1.1 Tariff measures

In assessing the robustness of our results we first turn our attention to the tariff data. The U.S. Census data on imports in 2001 consists of 251,920 observations. The largest sample we use in Table 2 consists of 45,918 observations. Some 95\% of the remaining 200,602 observations are not used due to them being associated with either zero tariffs or missing tariff information, while the remainder either do not have observations on fixed costs or transport costs or have unreasonably high transport costs. The large number of observations with zero tariffs is at odds with the observation that the U.S. Census data report positive duty collected for 150,688 observations or almost 60\%. This discrepancy likely stems from rules of origins not having been satisfied for some imports resulting in them receiving the MFN treatment, rather than the preferred duty-free entry or from some exporters not taking advantage of preferential access to the U.S., either out of ignorance or unwillingness to file the necessary paperwork.

In order to examine the sensitivity of our results to this behavior, we offer the two panels in Table 4 where we replicate the main regressions from Tables 2 and 3 with the elasticity of substitution as a measure of fixed costs, while using two different measures of tariffs. In the top panel we supplement the Romalis Tariff Database with U.S. 2001 MFN tariffs rates for every observation where our original tariff data report a tariff of zero and the Census data report positive duties collected. We do so under the assumption that if an exporter does not file the necessary paperwork to obtain preferential access to the U.S. market or does not satisfy rules of origin, their exports will receive MFN treatment. The bottom panel uses the Census data to calculate the average assessed tariff by taking a ratio of collected duties and the dutiable value of imports. The former approach increases our sample size to 68,162, and the latter to 120,541 observations, almost a half of all observed in 2001. In the latter approach the unused observations either have no duties collected resulting in a zero tariff or
Table 4: Different Measures of Tariffs

<table>
<thead>
<tr>
<th></th>
<th>All observations</th>
<th>High extensive margin</th>
<th>Low extensive margin</th>
<th>Total turnover</th>
<th>Net entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tariff rate</td>
<td>-0.371***</td>
<td>-0.798***</td>
<td>-0.605***</td>
<td>-0.341***</td>
<td>-0.344***</td>
</tr>
<tr>
<td></td>
<td>(0.06249)</td>
<td>(0.08509)</td>
<td>(0.08064)</td>
<td>(0.06209)</td>
<td>(0.06294)</td>
</tr>
<tr>
<td>Transport cost</td>
<td>-0.355***</td>
<td>-0.388***</td>
<td>-0.361***</td>
<td>-0.350***</td>
<td>-0.356***</td>
</tr>
<tr>
<td></td>
<td>(0.03026)</td>
<td>(0.04973)</td>
<td>(0.03954)</td>
<td>(0.03181)</td>
<td>(0.03227)</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>-0.071***</td>
<td>-0.441***</td>
<td>-0.220***</td>
<td>-0.045***</td>
<td>-0.045***</td>
</tr>
<tr>
<td></td>
<td>(0.01887)</td>
<td>(0.03427)</td>
<td>(0.03285)</td>
<td>(0.01822)</td>
<td>(0.01836)</td>
</tr>
<tr>
<td></td>
<td>(0.16083)</td>
<td>(0.23073)</td>
<td>(0.25005)</td>
<td>(0.15862)</td>
<td>(0.15941)</td>
</tr>
<tr>
<td>Observations</td>
<td>68,162</td>
<td>4,388</td>
<td>5,308</td>
<td>63,794</td>
<td>62,854</td>
</tr>
<tr>
<td>Number of country</td>
<td>200</td>
<td>166</td>
<td>165</td>
<td>199</td>
<td>198</td>
</tr>
<tr>
<td>R²</td>
<td>0.042</td>
<td>0.125</td>
<td>0.075</td>
<td>0.039</td>
<td>0.040</td>
</tr>
<tr>
<td>Test p-values</td>
<td>Result 1: $\theta = \zeta + \xi$</td>
<td>0.4298</td>
<td>0.7820</td>
<td>0.7981</td>
<td>0.4380</td>
</tr>
<tr>
<td></td>
<td>Result 2: $\theta = \zeta$</td>
<td>0.4030</td>
<td>0.0001</td>
<td>0.0065</td>
<td>0.5541</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>All observations</th>
<th>High extensive margin</th>
<th>Low extensive margin</th>
<th>Total turnover</th>
<th>Net entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tariff rate</td>
<td>-0.352***</td>
<td>-0.917***</td>
<td>-0.657***</td>
<td>-0.310***</td>
<td>-0.313***</td>
</tr>
<tr>
<td></td>
<td>(0.05292)</td>
<td>(0.07978)</td>
<td>(0.07652)</td>
<td>(0.05206)</td>
<td>(0.05259)</td>
</tr>
<tr>
<td>Transport cost</td>
<td>-0.325***</td>
<td>-0.298***</td>
<td>-0.341***</td>
<td>-0.321***</td>
<td>-0.323***</td>
</tr>
<tr>
<td></td>
<td>(0.03047)</td>
<td>(0.03739)</td>
<td>(0.03162)</td>
<td>(0.03226)</td>
<td>(0.03282)</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>-0.060***</td>
<td>-0.389***</td>
<td>-0.261***</td>
<td>-0.031*</td>
<td>-0.029</td>
</tr>
<tr>
<td></td>
<td>(0.01944)</td>
<td>(0.03126)</td>
<td>(0.02983)</td>
<td>(0.01824)</td>
<td>(0.01784)</td>
</tr>
<tr>
<td>Constant</td>
<td>12.286***</td>
<td>13.343***</td>
<td>13.239***</td>
<td>12.213***</td>
<td>12.196***</td>
</tr>
<tr>
<td></td>
<td>(0.12369)</td>
<td>(0.15742)</td>
<td>(0.13244)</td>
<td>(0.12576)</td>
<td>(0.12774)</td>
</tr>
<tr>
<td>Observations</td>
<td>120,541</td>
<td>7,110</td>
<td>8,770</td>
<td>113,431</td>
<td>111,771</td>
</tr>
<tr>
<td>Number of country</td>
<td>203</td>
<td>180</td>
<td>178</td>
<td>203</td>
<td>203</td>
</tr>
<tr>
<td>R²</td>
<td>0.039</td>
<td>0.156</td>
<td>0.101</td>
<td>0.035</td>
<td>0.035</td>
</tr>
<tr>
<td>Test p-values</td>
<td>Result 1: $\theta = \zeta + \xi$</td>
<td>0.5938</td>
<td>0.0328</td>
<td>0.5366</td>
<td>0.4763</td>
</tr>
<tr>
<td></td>
<td>Result 2: $\theta = \zeta$</td>
<td>0.3108</td>
<td>0.0000</td>
<td>0.0004</td>
<td>0.5857</td>
</tr>
</tbody>
</table>

The dependent variable is log of imports. Country fixed effects are included, robust standard errors clustered on countries are in parenthesis with *, **, *** denoting significance at 10%, 5%, and 1%.

All results in Table 4 are qualitatively similar to our earlier results. This is particularly reassuring in the case of results in the bottom panel where we use assessed tariff rather than posted ad valorem tariffs. These results indicate that in instances where posted tariffs are not available, assessed tariffs available in the data are a valid substitute.
4.1.2 Imports exclusive of tariffs and import charges

Our model above is derived for imports inclusive of all tariff and import charges, the latter of which reflect the cost of insurance and freight. The qualitative predictions of our model also hold if we use imports exclusive of tariff and import charges. The derivation of the model under such a condition can be found in an online appendix. In Table 5 we provide the same regressions as in Table 4 but using import values exclusive of tariff and import charges. Our results are qualitatively unchanged, with all estimated elasticity somewhat larger, and both relationships among elasticities holding according to the model.

<table>
<thead>
<tr>
<th>Table 5: Imports exclusive of tariff and import charges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Tariff rate</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Transport cost</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Fixed cost</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Constant</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>Number of countries</td>
</tr>
<tr>
<td>R²</td>
</tr>
<tr>
<td>Test p-values</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

The dependent variable is log of imports. Country fixed effects are included, robust standard errors clustered on countries are in parenthesis with *, **, *** denoting significance at 10%, 5%, and 1%.

5 Conclusion

In this paper, we took seriously the fact that variable costs based on value (ad valorem tariff) are fundamentally different than variable costs based on quantity (iceberg) in the context of a gravity model. We have presented a highly tractable model that allows for all three types of costs (two variable and one fixed). We have two main results in which we find empirical

\[ \vartheta = \frac{\sigma}{\sigma - 1}\zeta + 1, \quad \zeta = \gamma + 1, \quad \xi = \frac{\sigma}{\sigma - 1} - 1. \]

It can be readily seen that our three results hold in this case as well.

In the online appendix, we show that $\vartheta = \frac{\sigma}{\sigma - 1}\zeta + 1, \quad \zeta = \gamma + 1, \quad \xi = \frac{\sigma}{\sigma - 1} - 1$. It can be readily seen that our three results hold in this case as well.
support. The first is the linear relationship between the three elasticities; i.e. the sum of the elasticities of trade with respect to fixed and iceberg transport cost is equal to that of ad valorem tariffs. The second result is that the elasticity of trade with respect to tariffs is greater in magnitude than the elasticity of trade with respect to iceberg transport costs. This latter result is driven by the extensive margin both in the theory and empirical testing. We additionally show that if the variable cost is based on value (e.g. ad valorem tariff), then the elasticity of substitution plays a role in the overall trade elasticity but leave the testing of this claim to future research.

References


A Mathematical Appendix

A.1 Value of Exports Inclusive of Trade Costs

In this section we derive the gravity equation and elasticities under the case when the value of exports are calculated with tariffs or other trade costs included. To begin note that

\[
x_{ij}(\varphi) = \begin{cases} 
\lambda_3 \left( \frac{Y_i}{Y_j} \right)^{(\sigma-1)} \left( \frac{\theta_j}{w_i t_{ij}} \right)^{\sigma-1} \varphi^{\sigma-1}, & \text{if } \varphi \geq \varphi_{ij} \\
0 & \text{otherwise},
\end{cases} \tag{A-1}
\]

\[
\varphi_{ij} = \lambda_4 \left( \frac{Y}{Y_j} \right)^{\frac{1}{\gamma}} \left( \frac{w_i \tau_{ij}}{\theta_j} \right)^{\frac{1}{\sigma-1}} \left( f_{ij} t_{ij}^\sigma \right)^{\frac{1}{\sigma-1}} \\
Y_i = (1 + \lambda_5) w_i L_i \\
\pi = \lambda_5
\]

where \(\lambda_3, \lambda_4,\) and \(\lambda_5\) are constants.\textsuperscript{17}

A.1.1 Deriving the Gravity Equation

Aggregate Exports in sector \(h\) from country \(i\) to country \(j\) is

\[
X^h_{ij} = w_i L_i \int_{\varphi_{ij}}^{\infty} x^h_{ij}(\varphi) dG(\varphi).
\]

Using the specific assumption about the distribution \(G\), this becomes

\[
X^h_{ij} = w_i L_i \int_{\varphi_{ij}}^{\infty} \lambda_3^h \left( \frac{Y_i}{Y} \right)^{(\sigma-1)} \left( \frac{\theta_j}{w_i t_{ij}} \right)^{\sigma-1} \varphi^{\sigma-1} \left( f_{ij} t_{ij}^\sigma \right)^{\frac{1}{\sigma-1}} \frac{1}{\gamma} \varphi^{-\gamma - 1} d\varphi.
\]

\[
\lambda_3 = \sigma \lambda_4^{1-\sigma} \\
\lambda_4 = \left[ \left( \frac{\sigma}{\mu_j} \right) \left( \frac{\gamma}{\gamma - (\sigma - 1)} \right) \frac{1}{(1 + \lambda_5)} \right]^\frac{\gamma}{\gamma - (\sigma - 1)} \\
\lambda_5 = \frac{\sum_{h=1}^{H} \left( \frac{\sigma_h - 1}{\gamma_h} \right) \frac{\varphi_h}{\sigma_h}}{1 - \sum_{h=1}^{H} \left( \frac{\sigma_h - 1}{\gamma_h} \right) \frac{\varphi_h}{\sigma_h}}
\]
Solving this integral yields:

\[
X_{ij}^h = \left( \frac{Y_j}{Y} \right)^{\frac{(\sigma_h-1)}{\gamma_h}} \left( \frac{\theta_j}{\gamma_h} \right)^{\frac{\sigma_h-1}{\gamma_h}} \left( \frac{w_i L_i \lambda^h_{ij}}{\gamma_h - (\sigma_h - 1)} \right) \left[ \lambda^{h} \left( \frac{Y_j}{Y} \right) \frac{\theta_j}{\gamma_h - (\sigma_h - 1)} \right] \left[ \int_{0}^{\infty} \left( \frac{w_i \tau_{ij}^h}{\theta_j} \right)^{\frac{(\sigma_h-1)}{\gamma_h}} f_{ij}^{\frac{(\sigma_h-1)}{\gamma_h}} \right]^{\frac{(\sigma_h-1)}{\gamma_h}}
\]

\[
= \frac{w_i L_i \lambda^h_{ij}}{Y} \left( \frac{Y_j}{Y} \right)^{1 - \frac{\sigma_h \gamma_h}{\gamma_h - (\sigma_h - 1)}} \left( \frac{\theta_j}{\gamma_h - (\sigma_h - 1)} \right) \left( \frac{w_i \tau_{ij}^h}{\theta_j} \right)^{\frac{\gamma_h}{\gamma_h - (\sigma_h - 1)}} f_{ij}^{\frac{(\sigma_h-1)}{\gamma_h}}
\]

\[
= \frac{w_i L_i \lambda^h_{ij}}{Y} \left( \frac{Y_j}{Y} \right)^{1 - \frac{\sigma_h \gamma_h}{\gamma_h - (\sigma_h - 1)}} \left( \frac{w_i \tau_{ij}^h}{\theta_j} \right)^{\frac{\gamma_h}{\gamma_h - (\sigma_h - 1)}} f_{ij}^{\frac{(\sigma_h-1)}{\gamma_h}}
\]

Therefore, total \( X_{ij}^h \) in sector \( h \) from country \( i \) to country \( j \) are given by

\[
X_{ij}^h = \mu_h \left( \frac{Y_i Y_j}{Y} \right) \left( \frac{w_i \tau_{ij}^h}{\theta_j} \right)^{\frac{\gamma_h}{\gamma_h - (\sigma_h - 1)}} f_{ij}^{\frac{(\sigma_h-1)}{\gamma_h}}.
\] (A.2)

**A.1.2 Deriving Elasticities**

Totally differentiating (A.2) for a specific sector \( h \) and assuming \( df_{ij} = 0 \) yields the following elasticities:

\[
- \frac{d \ln X_{ij}^h}{d \ln \tau_{ij}} = \frac{-d X_{ij}^h/d \tau_{ij}}{X_{ij}^h/\tau_{ij}} = \frac{-\tau_{ij}}{X_{ij}^h} \left( \frac{w_i L_i}{X_{ij}^h} \int_{\bar{\varphi}_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial \varphi_{ij}} dG(\varphi) \right) + \frac{\tau_{ij}}{X_{ij}^h} \left( \frac{w_i L_i x(\varphi_{ij}) G(\varphi_{ij})}{X_{ij}^h} \frac{\partial \varphi_{ij}}{\partial \tau_{ij}} \right)
\]

**Intensive margin**

\[
- \frac{d \ln X_{ij}^h}{d \ln \tau_{ij}} = \frac{-d X_{ij}^h/d \tau_{ij}}{X_{ij}^h/\tau_{ij}} = \frac{-\tau_{ij}}{X_{ij}^h} \left( \frac{w_i L_i}{X_{ij}^h} \int_{\bar{\varphi}_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial \varphi_{ij}} dG(\varphi) \right) + \frac{\tau_{ij}}{X_{ij}^h} \left( \frac{w_i L_i x(\varphi_{ij}) G(\varphi_{ij})}{X_{ij}^h} \frac{\partial \varphi_{ij}}{\partial \tau_{ij}} \right)
\]

**Extensive margin**

Using the definition of equilibrium individual exports from equation (A.1), and assuming that country \( i \) is small enough and/or remote enough, so that \( \partial \theta_j/\partial t_{ij} \approx 0 \) and \( \partial \theta_j/\partial \tau_{ij} \approx 0 \), we get

\[
\frac{\partial x_{ij}(\varphi)}{\partial t_{ij}} = -(\sigma - 1) \frac{x_{ij}(\varphi)}{t_{ij}} \quad \text{and} \quad \frac{\partial x_{ij}(\varphi)}{\partial \tau_{ij}} = -(\sigma - 1) \frac{x_{ij}(\varphi)}{\tau_{ij}}.
\]
Integrating over all exporters, we get

$$Elasticity \text{ of the intensive margin w.r.t. tariffs} = -\frac{t_{ij}}{X_{ij}} \left( w_i L_i \int_{\phi_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial t_{ij}} dG(\varphi) \right)$$

$$= (\sigma - 1) \frac{t_{ij} w_i L_i \int_{\phi_{ij}}^{\infty} x_{ij}(\varphi) dG(\varphi)}{t_{ij}}$$

$$= (\sigma - 1) \frac{t_{ij} X_{ij}}{t_{ij}}$$

$$= (\sigma - 1).$$

Now, define $x_{ij} = \lambda_{ij} \varphi^{\sigma - 1}$ and note that $G'(\varphi) = \varphi^{-\gamma - 1}/\gamma$.Aggregate exports can be written in the following way

$$X_{ij} = w_i L_i \lambda_{ij} \int_{\phi_{ij}}^{\infty} \varphi^{\sigma - 1} \gamma \varphi^{-\gamma - 1}$$

$$= \frac{\gamma}{\gamma - (\sigma - 1)} w_i L_i \lambda_{ij} \varphi^{(\sigma - 1) - \gamma}$$

$$= \frac{1}{\gamma - (\sigma - 1)} w_i L_i x_{ij}(\bar{\varphi}) G'(\bar{\varphi}) \bar{\varphi}.$$

We therefore get the simple solution for the elasticity:

$$Elasticity \text{ of the extensive margin w.r.t. tariffs} = \frac{t_{ij}}{X_{ij}} \left( w_i L_i x_{ij}(\bar{\varphi}) G'(\bar{\varphi}) \frac{\partial \bar{\varphi}}{\partial t_{ij}} \right)$$

$$= \frac{t_{ij}}{X_{ij}} \left( \frac{w_i L_i x_{ij}(\bar{\varphi}) G'(\bar{\varphi}) \bar{\varphi}}{t_{ij}} \right)$$

$$= \frac{(\gamma - (\sigma - 1)) t_{ij}}{X_{ij}} \left( \frac{X_{ij}}{t_{ij} \sigma - 1} \right)$$

$$= \frac{\sigma \gamma}{\sigma - 1} - \sigma.$$

Similarly for iceberg transport costs:

$$Elasticity \text{ of the intensive margin w.r.t. iceberg costs} = -\frac{\tau_{ij}}{X_{ij}} \left( w_i L_i \int_{\phi_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial \tau_{ij}} dG(\varphi) \right)$$

$$= (\sigma - 1) \frac{\tau_{ij} w_i L_i \int_{\phi_{ij}}^{\infty} x_{ij}(\varphi) dG(\varphi)}{\tau_{ij}}$$

$$= (\sigma - 1) \frac{\tau_{ij} X_{ij}}{\tau_{ij}}$$

$$= (\sigma - 1).$$
Elasticity of the extensive margin w.r.t. tariffs  

\[
\frac{\tau_{ij}}{X_{ij}} \left( \frac{w_i L_i x_{ij}(\bar{\phi}) G'(\bar{\phi})}{X_{ij}} \right) = (\gamma - (\sigma - 1)) \frac{\tau_{ij}}{X_{ij}} \left( \frac{X_{ij}}{\tau_{ij}} \right) = (\gamma - (\sigma - 1)).
\]

The elasticity with respect to fixed costs is identical to that of Chaney (2008). Thus we have

\[
\text{Tariiff: } \vartheta \equiv -\frac{d \ln X_{ij}}{d \ln t_{ij}} = (\sigma - 1) + \frac{\sigma \gamma}{\sigma - 1} - \sigma = \frac{\sigma \gamma - 1}{\sigma - 1}, \quad (A-3)
\]

\[
\text{Iceberg: } \zeta \equiv -\frac{d \ln X_{ij}}{d \ln \tau_{ij}} = (\sigma - 1) + [\gamma - (\sigma - 1)] = \gamma \quad (A-4)
\]

\[
\text{Fixed Cost: } \xi \equiv -\frac{d \ln X_{ij}}{d \ln f_{ij}} = 0 + \frac{\gamma}{\sigma - 1} - 1 = \frac{\gamma}{\sigma - 1} - 1. \quad (A-5)
\]

Our three main results follow from inspection of the trade elasticities.

**Result A-1.** The elasticity with respect to tariffs is equal to the sum of the elasticity with respect to fixed and iceberg costs:

\[
\xi + \zeta = \vartheta.
\]

**Result A-2.** Trade flows are more elastic with respect to changes in tariffs than transport costs,

\[
\vartheta - \zeta = \frac{\gamma - (\sigma - 1)}{\sigma - 1} > 0.
\]

**Result A-3.** The elasticity of trade flows with respect to ad valorem tariffs is decreasing in the elasticity of substitution,

\[
\frac{d \vartheta}{d \sigma} = \frac{-\gamma}{(\sigma - 1)^2} < 0.
\]
A.2 Value of Exports Exclusive of Trade Costs

In this section we derive the gravity equation and elasticities under the case when the value of exports are calculated without tariffs or other trade costs. To begin note that

\[ x_{ij}(\varphi) = \begin{cases} 
\lambda_3 \left( \frac{Y_j}{Y_i} \right)^{\frac{(\sigma-1)}{\gamma}} \tau_{ij}^{-\sigma} t_{ij}^{-\sigma} \left( \frac{\theta_i}{\mu_i} \right)^{\sigma-1} \varphi^{\sigma-1}, & \text{if } \varphi \geq \tilde{\varphi}_{ij} \\
0 & \text{otherwise,} \end{cases} \]  

(A-6)

\[ \tilde{\varphi}_{ij} = \lambda_4 \left( \frac{Y_j}{Y_i} \right)^{\frac{1}{3}} \left( \frac{w_i \tau_{ij}}{\theta_j} \right) \left( \frac{f_{ij} t_{ij}^{\sigma}}{\beta_j} \right)^{\frac{1}{(\sigma-1)}} \]

\[ Y_i = (1 + \lambda_5) w_i L_i \]

\[ \pi = \lambda_5 \]

where \( \lambda_3, \lambda_4, \) and \( \lambda_5 \) are constants.\[18\]

A.2.1 Deriving the Gravity Equation

Aggregate Exports in sector \( h \) from country \( i \) to country \( j \) is

\[ X_{ij}^h = w_i L_i \int_{\tilde{\varphi}_{ij}}^{\infty} x_{ij}^h(\varphi) dG(\varphi). \]

Using the specific assumption about the distribution \( G \), this becomes

\[ X_{ij}^h = w_i L_i \int_{\tilde{\varphi}_{ij}}^{\infty} \lambda_3^h \left( \frac{Y_j}{Y_i} \right)^{\frac{(\sigma_h-1)}{\gamma_h}} \left( \tau_{ij}^{h} t_{ij}^{h} \right)^{\frac{1}{(\gamma_h - (\sigma_h - 1))}} \left( \frac{\beta_j}{w_i} \right)^{\sigma_h-1} \varphi^{\sigma_h-1} \frac{\varphi^{-\gamma_h-1}}{\gamma_h} d\varphi. \]

\[ \lambda_3 = \sigma \lambda_4^{1-\sigma} \]

\[ \lambda_4 = \left[ \left( \frac{\sigma}{\mu} \right) \left( \frac{\gamma}{\gamma - (\sigma - 1)} \right) \left( 1 + \lambda_5 \right) \right]^{\frac{1}{\gamma}} \]

\[ \lambda_5 = \frac{\sum_{h=1}^{H} \left( \frac{\sigma_h-1}{\gamma_h} \right) \mu_h}{1 - \sum_{h=1}^{H} \left( \frac{\sigma_h-1}{\gamma_h} \right) \mu_h} \]
Solving this integral yields:

\[ X^h_{ij} = \left( \frac{Y_j}{Y} \right)^{\frac{(\sigma_h-1)}{\gamma_h}} \left( \frac{\theta_j}{w_i} \right)^{\sigma_h-1} \frac{w_i L_i \lambda_i^h}{\gamma_h - (\sigma_h - 1)} \left[ \lambda_i^h \left( \frac{Y_j}{Y} \right)^{\frac{1}{\gamma_h}} \left( \frac{w_i t_{ij}^h}{\theta_j} \right)^{-\frac{1}{\sigma_h-1}} \right] \left( \frac{Y_j}{Y} \right)^{\frac{(\sigma_h-1)-\gamma_h}{\gamma_h - (\sigma_h - 1)}} \]

\[ = w_i L_i \lambda_i^h \left( \frac{Y_j}{Y} \right)^{\frac{t_{ij}^h}{\theta_i}} \left( \frac{t_{ij}^h}{\gamma_h - (\sigma_h - 1)} \right) \left( \frac{w_i}{\theta_j} \right)^{-\frac{1}{\sigma_h-1}} f_{ij} \]

\[ = \lambda_i^h (\sigma_h-1)^{\gamma_h} \left( \frac{w_i L_i Y_j}{Y} \right)^{\frac{t_{ij}^h}{\gamma_h - (\sigma_h - 1)}} \left( \frac{w_i}{\theta_j} \right)^{-\frac{1}{\sigma_h-1}} f_{ij} \]

\[ = \mu_h (1 + \lambda_i^h) \left( \frac{w_i L_i Y_j}{Y} \right)^{\frac{t_{ij}^h}{\gamma_h - (\sigma_h - 1)}} \left( \frac{w_i}{\theta_j} \right)^{-\frac{1}{\sigma_h-1}} f_{ij} \]

\[ = \mu_h \left( \frac{Y_j Y_i}{Y} \right)^{\frac{t_{ij}^h}{\gamma_h - (\sigma_h - 1)}} \left( \frac{w_i}{\theta_j} \right)^{-\frac{1}{\sigma_h-1}} f_{ij} \]

Therefore, total (f.o.b.) \( X^h_{ij} \) in sector \( h \) from country \( i \) to country \( j \) are given by

\[ X^h_{ij} = \mu_h \left( \frac{Y_j Y_i}{Y} \right)^{\frac{t_{ij}^h}{\gamma_h - (\sigma_h - 1)}} \left( \frac{w_i}{\theta_j} \right)^{-\frac{1}{\sigma_h-1}} f_{ij}^{\frac{\gamma_h}{(\gamma_h - (\sigma_h - 1))}} \] (A-7)

A.2.2 Deriving Elasticities

Totally differentiating (A-7) for a specific sector \( h \) and assuming \( df_{ij} = 0 \) yields the following elasticities:

\[ - \frac{d \ln X_{ij}}{d \ln t_{ij}} = - \frac{d X_{ij}/dt_{ij}}{X_{ij}/t_{ij}} = - \frac{t_{ij}}{X_{ij}} \left( \frac{w_i L_i x(\varphi_{ij}) G'(\varphi_{ij}) \partial \varphi_{ij}}{\partial t_{ij}} \right) \]

Intensive margin

\[ - \frac{d \ln X_{ij}}{d \ln \tau_{ij}} = - \frac{d X_{ij}/d\tau_{ij}}{X_{ij}/\tau_{ij}} = - \frac{\tau_{ij}}{X_{ij}} \left( \frac{w_i L_i x(\varphi_{ij}) G'(\varphi_{ij}) \partial \varphi_{ij}}{\partial \tau_{ij}} \right) \]

Intensive margin

Using the definition of equilibrium individual exports from equation (A-7), and assuming that country \( i \) is small enough and/or remote enough, so that \( \partial \theta_j / \partial t_{ij} \approx 0 \) and \( \partial \theta_j / \partial \tau_{ij} \approx 0 \),
I get
\[ \frac{\partial x_{ij}(\varphi)}{\partial t_{ij}} = -\sigma \frac{x_{ij}(\varphi)}{t_{ij}} \quad \text{and} \quad \frac{\partial x_{ij}(\varphi)}{\partial \tau_{ij}} = -\sigma \frac{x_{ij}(\varphi)}{\tau_{ij}}. \]

Integrating over all exporters, we get

\[
\text{Elasticity of the intensive margin w.r.t. tariffs} = \frac{t_{ij} w_i L_i}{X_{ij}} \left( w_i L_i \int_{\varphi_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial t_{ij}} dG(\varphi) \right) = \sigma \frac{t_{ij} w_i L_i}{X_{ij}} \int_{\varphi_{ij}}^{\infty} x_{ij}(\varphi) dG(\varphi) = \sigma \frac{t_{ij} X_{ij}}{t_{ij}} = \sigma. \]

Now, define \( x_{ij} = \lambda_{ij} \varphi^{\sigma-1} \) and note that \( G'(\varphi) = \varphi^{-1}/\gamma \). Aggregate exports can be written in the following way

\[
X_{ij} = w_i L_i \lambda_{ij} \int_{\varphi_{ij}}^{\infty} \varphi^{\sigma-1} \varphi^{-\gamma-1} = \frac{\gamma}{\gamma - (\sigma - 1)} w_i L_i \lambda_{ij} \varphi^{(\sigma-1)-\gamma} = \frac{1}{\gamma - (\sigma - 1)} w_i L_i x_{ij}(\bar{\varphi}) G'(\bar{\varphi}) \bar{\varphi}.
\]

I therefore get the simple solution for the elasticity:

\[
\text{Elasticity of the extensive margin w.r.t. tariffs} = \frac{t_{ij} w_i L_i x_{ij}(\bar{\varphi}) G'(\bar{\varphi})}{X_{ij}} \left( \frac{\partial \bar{\varphi}}{\partial t_{ij}} \right) = \frac{t_{ij} w_i L_i x_{ij}(\bar{\varphi}) G'(\bar{\varphi})}{X_{ij}} \left( \frac{\varphi}{\sigma - 1} \right) = (\gamma - (\sigma - 1)) \frac{t_{ij} X_{ij}}{t_{ij} \sigma} = \frac{\sigma}{\sigma - 1} - \sigma.
\]

Similarly for iceberg transport costs:

\[
\text{Elasticity of the intensive margin w.r.t. iceberg costs} = -\frac{\tau_{ij} w_i L_i}{X_{ij}} \left( w_i L_i \int_{\varphi_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial \tau_{ij}} dG(\varphi) \right) = \sigma \frac{\tau_{ij} w_i L_i}{X_{ij}} \int_{\varphi_{ij}}^{\infty} x_{ij}(\varphi) dG(\varphi) = \sigma \frac{\tau_{ij} X_{ij}}{\tau_{ij}} = \sigma.
\]
Furthermore, the extensive margin is as follows:

\[ \text{Elasticity of the extensive margin w.r.t. tariffs} = \frac{\tau_{ij}}{X_{ij}} \left( L_i x_{ij}(\tilde{\phi}) G'(\tilde{\phi}) \frac{\partial \tilde{\phi}}{\partial \tau_{ij}} \right) \]

\[ = \frac{\tau_{ij}}{X_{ij}} \left( L_i x_{ij}(\tilde{\phi}) G'(\tilde{\phi}) \tilde{\phi} \right) \]

\[ = (\gamma - (\sigma - 1)) \frac{\tau_{ij}}{X_{ij}} \left( \frac{X_{ij}}{\tau_{ij}} \right) \]

\[ = (\gamma - (\sigma - 1)). \]

The elasticity with respect to fixed costs is identical to that of Chaney (2008). Thus we have

\[ \text{Tariff:} \quad \rightarrow \vartheta \equiv -\frac{d\ln X_{ij}}{d\ln t_{ij}} = \underbrace{\frac{\sigma}{\sigma - 1}} + \frac{\sigma \gamma}{\sigma - 1} - \sigma = \frac{\sigma \gamma}{\sigma - 1}, \quad (A-8) \]

\[ \text{Intensive} \quad \text{Extensive} \]

\[ \text{Iceberg:} \quad \rightarrow \zeta \equiv -\frac{d\ln X_{ij}}{d\ln \tau_{ij}} = \underbrace{\frac{\sigma}{\sigma - 1}} + \left[ \frac{\gamma - (\sigma - 1)}{\sigma - 1} \right] = \gamma + 1 \quad (A-9) \]

\[ \text{Intensive} \quad \text{Extensive} \]

\[ \text{Fixed Cost:} \quad \rightarrow \xi \equiv -\frac{d\ln X_{ij}}{d\ln f_{ij}} = \underbrace{0} + \frac{\gamma}{\sigma - 1} - 1 = \frac{\gamma}{\sigma - 1} - 1. \quad (A-10) \]

\[ \text{Intensive} \quad \text{Extensive} \]

Our three main results follow from inspection of the trade elasticities and are identical to the previous section when trade costs were included in the value of exports.

**Result A-1.** The elasticity with respect to tariffs is equal to the sum of the elasticity with respect to fixed and iceberg costs:

\[ \xi + \zeta = \vartheta. \]

**Result A-2.** Trade flows are more elastic with respect to changes in tariffs than transport costs,

\[ \vartheta - \zeta = \frac{\gamma - (\sigma - 1)}{\sigma - 1} > 0. \]

**Result A-3.** The elasticity of trade flows with respect to ad valorem tariffs is decreasing in the elasticity of substitution,

\[ \frac{d\vartheta}{d\sigma} = \frac{-\gamma}{(\sigma - 1)^2} < 0. \]